Application of Firefly Algorithm to Gaussian Process-based Prediction of Electric Power Damage Caused by Typhoons

Tomohiro Hachino, Hitoshi Takata, Shigeru Nakayama, Seiji Fukushima, and Yasutaka Igarashi

Abstract—Firefly algorithm (FA) is an optimization technique inspired by an intelligent behavior of firefly swarms. This paper presents an application of FA to Gaussian process (GP)-based prediction of electric power damage caused by typhoons. The GP prior model for prediction is trained by a separable least-squares (LS) approach combining the linear LS method with FA. The proposed prediction system yields not only the predicted amount of damage but also its confidence measure. Simulation results based on actual data of typhoons that hit or came close to the Amami archipelago in Japan are shown to illustrate the effectiveness of this prediction system.

Keywords—Damage caused by typhoon, electric power system, firefly algorithm, Gaussian process model, prediction.

I. I NTRODUCTION

D AMAGE to electric power facilities caused by typhoons is one of the most common meteorological disasters in Japan [1]–[3]. Once the electric power supply is cut off by typhoons, the social life is paralyzed. It is an urgent work to restore the electric power supply and to resume normal services as soon as possible. To ensure the speedy restoration of the electric power supply, an accurate prediction of the amount of damage is required, so that the staff and materials necessary for restoration could be appropriately arranged. This is particularly important for isolated island areas, because the staff and materials necessary for restoration must be dispatched there just before ships and airplanes are canceled due to the typhoon.

In the field, empirical predictions based on past typhoon weather information and electric power damage have been utilized. However, it is difficult for such predictions to keep the objectivity. On the other hand, the authors have presented two-stage predictors that consist of neural networks and linear or second-order regression from the viewpoint of nonlinear prediction [4], [5]. However, these prediction methods need to use a large number of parameters to describe the nonlinearity between the typhoon weather information and the electric power damage. This is presently one of the drawbacks of these predictors, because we can use only limited amounts of training input (typhoon weather information) and output (electric power damage) data. Moreover, confidence measures for the predicted amount of damage cannot be obtained for the two-stage predictors.

In this paper, we present an application of firefly algorithm (FA) [6] to Gaussian process (GP)-based prediction of electric power damage caused by typhoons [7]. The GP model is a nonparametric model and fits naturally into the Bayesian framework [8]–[10]. This model has recently attracted much attention for system identification [11], [12], time series forecasting [13], [14], and predictive control [15]. The proposed GP-based predictor includes far fewer parameters to describe the nonlinearity than the two-stage predictors. The predicted amount of damage is given by the predictive mean of the GP and its confidence measure is evaluated by the predictive variance of the GP.

The parameters included in the GP model and the adjusting parameters for quantification of the typhoon track have to be properly trained based on the training input and output data. Generally this training becomes nonlinear optimization problem. In this paper, a separable least-squares (LS) approach combining the linear LS method with FA is presented for this training. FA is an optimization algorithm inspired by an intelligent behavior of firefly swarms [6]. In FA, for any two flashing fireflies, the less brighter firefly moves toward the brighter one according to the attractiveness. The attractiveness is proportional to the light intensity observed by the partner and monotonically decreases as the distance between two fireflies increases, owing to the inverse square law and the absorption property of light. FA consists of only the basic arithmetic operations and does not require complicated coding and genetic operations such as crossovers and mutations.
of genetic algorithm (GA). Moreover, the performance and computational cost of FA are shown to be better than those of other population-based algorithms such as GA and particle swarm optimization (PSO) [6], [16]. These advantages suggest that the use of FA increases efficiency when the GP-based predictor is trained.

This paper is organized as follows. In Section II, the problem is formulated. In Section III, the quantification technique for the typhoon track is described. In Section IV, the GP prior model for prediction is derived. In Section V, the separable LS approach using FA is presented for training of the GP prior model. In Section VI, the prediction of the electric power damage is carried out using the GP posterior distribution. In Section VII, the performance of the proposed prediction system is demonstrated through numerical simulation using actual data of damage for the Amami archipelago in Japan. Finally, conclusions are given in Section VIII.

II. STATEMENT OF THE PROBLEM

The objective area for prediction is taken to be the Amami archipelago in Japan. This archipelago is located at about latitude 27.83°N and longitude 128.08°E.

The input of the predictor is the typhoon weather information:

\[ \mathbf{x} = [x_1, x_2]^T \]  

where \( x_1 \) is the typhoon track and \( x_2 \) [m/s] is the maximum instantaneous wind speed. The output from the predictor is the electric power damage \( y \), such as the power failure circuit. It is possible to choose other weather information as the input, but it increases the scale of the predictor. Therefore, we choose only the typhoon track and the maximum instantaneous wind speed that affect the amount of electric power damage greatly.

It is assumed that we collect the typhoon weather data released from the Meteorological Agency:

\[ \mathbf{X} = [\mathbf{x}(1), \mathbf{x}(2), \ldots, \mathbf{x}(N)]^T, \]
\[ \mathbf{x}(j) = [x_1(j), x_2(j)]^T \]  

and the corresponding actual data of the amount of electric power damage:

\[ \mathbf{y} = [y(1), y(2), \ldots, y(N)]^T \]

where \( N \) is the number of the typhoons that hit or came close to the Amami archipelago in the past.

The purpose of this paper is to construct a prediction system that can predict the amount of electric power damage with its confidence measure from the weather data of a new approaching typhoon.

III. QUANTIFICATION OF TYphoon TRACK

The typhoon track strongly correlates with the amount of electric power damage. In order to input the typhoon track into the predictor, we have to quantify it as a numerical value. In general, in the Northern Hemisphere, the wind force in the east side of the typhoon is stronger than that in the west side of the typhoon. This wind characteristic suggests that the typhoon via the west side of the Amami archipelago probably causes more damage than the typhoon via the east side of the Amami archipelago. Moreover, since the typhoon is likely to stay around the Amami archipelago for a long time, the electric power system may frequently suffer from major damage. Therefore, we have to consider the wind characteristic and the stagnancy of the typhoon when the typhoon track is quantified. First, the centers of the typhoon are plotted every hour in the range from latitude 26°N to 31°N. Then a Gaussian function is arranged on the Amami archipelago as shown in Fig. 1.

The numerical value of the typhoon track is calculated by summing the altitude values of the arranged function corresponding to the plotted centers as follows:

\[ x_1 = \sum_{j=1}^{n} z_j \]
\[ z_j = \exp \left\{ \frac{- (T_{LAj} - C_{LA})^2 + (T_{LOj} - C_{LO} + \alpha)^2}{\beta^2} \right\} \]

where \( T_{LAj} \) is the latitude of the typhoon center, \( T_{LOj} \) is the longitude of the typhoon center, \( C_{LA} \) is the latitude of the Amami archipelago, \( C_{LO} \) is the longitude of the Amami archipelago, \( \alpha > 0 \) is the bias for the typhoon center, \( \beta \) is the width of the Gaussian function, and \( n \) is the number of the plotted centers of the typhoon. Note that the value of the typhoon track becomes large in the case that the typhoon stays around the Amami archipelago for a long time. The bias \( \alpha \) is introduced to take the wind characteristic of the typhoon into consideration. A way of determining the adjusting parameter vector \( \theta_p = [\alpha, \beta]^T \) suboptimally will be discussed in Section V.
IV. GP PRIOR MODEL FOR PREDICTION

Assume that the relation between the typhoon weather information \( x \) and the amount of electric power damage \( y \) is described as

\[
y = f(x) + \varepsilon
\]  
(5)

where \( f(\cdot) \) is a function which is assumed to be stationary and smooth. \( \varepsilon \) is assumed to be a zero-mean Gaussian noise with variance \( \sigma_{n}^{2} \).

Let the function value vector corresponding to the typhoon weather data given by (2) be

\[
f = [f(x(1)), f(x(2)), \cdots, f(x(N))]^T
\]  
(6)

Then this function value vector \( f \) is represented by GP regression. The GP is a Gaussian random function and is completely described by its mean function and covariance function. We can regard it as a collection of random variables with a joint multivariable Gaussian distribution. Therefore, the function value vector \( f \) can be represented by the GP as

\[
f \sim N(m(X), \Sigma(X, X))
\]  
(7)

where \( m(X) \) is the \( N \)-dimensional mean function vector and \( \Sigma(X, X) \) is the \( N \)-dimensional covariance matrix evaluated at all pairs of the training input data. Equation (7) means that \( f \) has a Gaussian distribution with the mean function vector \( m(X) \) and the covariance matrix \( \Sigma(X, X) \).

In this paper, the mean function \( m(x) \) is expressed by the first-order polynomial, i.e. a linear combination of the input variable:

\[
m(x) = x\theta_{m}
\]  
(8)

where \( x = [x^T, 1] \) and \( \theta_{m} = [\theta_{m1}, \theta_{m2}, \theta_{m3}]^T \) is the unknown weighting parameter vector for the mean function. Thus, the mean function vector \( m(X) \) is described as follows:

\[
m(X) = [m(x(1)), m(x(2)), \cdots, m(x(N))]^T
\]  
(9)

\[
= \bar{X}\theta_{m}
\]

where \( \bar{X} = [X, e] \) and \( e = [1, 1, \cdots, 1]^T \) is the \( N \)-dimensional vector of ones.

The covariance \( \Sigma_{pq}=\text{cov}(f(x(p), f(x(q))))=s(x(p), x(q)) \) is an element of the covariance matrix \( \Sigma(X, X) \), which is a function of \( x(p) \) and \( x(q) \). In this paper, the following Gaussian kernel is utilized as the covariance function \( s(x(p), x(q)) \):

\[
s(x(p), x(q)) = \sigma_{y}^{2}\exp\left(-\frac{\|x(p) - x(q)\|^2}{2\ell^{2}}\right)
\]  
(10)

where \( \| \cdot \| \) denotes the Euclidean norm. Equation (10) means that the covariance of the function values depends only on the distance between the inputs \( x(p) \) and \( x(q) \). A high correlation between the function values occurs for inputs that are close to each other. The overall variance of the random function can be controlled by varying \( \sigma_{y}^{2} \), and the characteristics length scale of the process can be changed by varying \( \ell \).

As the amount of electric power damage \( y \) is noisy observation, we can derive the following GP prior regression from (7):

\[
y \sim N(m(X), K(X, X))
\]  
(11)

where

\[
K(X, X) = \Sigma(X, X) + \sigma_{n}^{2}I_{N}
\]  
(12)

\[
I_{N} : N \times N \text{ identity matrix}
\]

and \( \theta_{c} = [\sigma_{y}, \ell, \sigma_{n}]^T \) is called the hyperparameter vector. In the following, \( K(X, X) \) is written as \( K \) for simplicity.

V. TRAINING BY FA

The accuracy of the prediction greatly depends on the unknown parameter vectors, i.e., the weighting parameter vector \( \theta_{m} \) of the mean function, the hyperparameter vector \( \theta_{c} \) of the covariance function, and the adjusting parameter vector \( \theta_{p} \) of the quantification of the typhoon track. Therefore, the parameter vector \( \theta = [\theta_{m}^T, \theta_{c}^T, \theta_{p}^T]^T \) has to be determined suboptimally. This training is carried out by maximizing the log marginal likelihood of the typhoon weather data and the actual amount of electric power damage:

\[
J = \log p(y | X, \theta)
\]

\[
= -\frac{1}{2} \log |K| - \frac{1}{2}(y - \bar{X}\theta_{m})^TK^{-1}(y - \bar{X}\theta_{m})
\]

\[
- \frac{N}{2} \log(2\pi)
\]

(13)

As the cost function \( J \) generally has multiple local maxima, this training becomes a nonlinear optimization problem. However, we can separate the linear optimization part and the nonlinear optimization part for this problem. Note that if the candidates for the hyperparameter vector \( \theta_{c} \) and adjusting parameter vector \( \theta_{p} \) are given, the weighting parameter vector \( \theta_{m} \) can be estimated by the linear LS method putting \( \partial J/\partial \theta_{m} = 0 \):

\[
\theta_{m} = (\bar{X}^TK^{-1}\bar{X})^{-1}\bar{X}^TK^{-1}y
\]  
(14)

However, even if \( \theta_{m} \) is known, the optimization with respect to \( \theta_{c} \) and \( \theta_{p} \) is a complicated nonlinear problem and might suffer from the local optima problem. Therefore, in this paper, we present the separable LS approach combining the linear LS method with FA to determine the unknown parameter vector \( \theta \). Only \( \Omega = [\theta_{m}^T, \theta_{c}^T]^T = [\sigma_{y}, \ell, \sigma_{n}, \alpha, \beta]^T \) is represented with the positions of fireflies in the search space and is searched for by FA. The detailed training algorithm is as follows:

**step 1: Initialization**

Generate an initial population of \( Q \) fireflies with random positions \( \Omega_{i} \) (\( i = 1, 2, \cdots, Q \)).

Set the iteration counter \( l \) to 0.

**step 2: Quantification of the typhoon track**

Quantify the typhoon track to the numerical value using \( \Omega_{p} \) (\( i = 1, 2, \cdots, Q \)) by the quantification technique given in Section III.

**step 3: Construction of the covariance matrix**
Construct $Q$ candidates of the covariance matrix $K_i$ using \( \theta_{c[i]} \) (\( i = 1, 2, \ldots, Q \)).

**step 4: Estimation of \( \theta_m \)**

Estimate \( Q \) candidates of \( \theta_{m[i]} \) (\( i = 1, 2, \ldots, Q \)) from (14).

**step 5: Light intensity calculation**

Calculate the light intensity \( I_i \) of each firefly from the log marginal likelihood of the typhoon weather data and the actual amount of electric power damage given by (13):

\[
I_i(\Omega_{[i]}) = -\frac{1}{2} \log \left| K_i \right| - \frac{1}{2} (y - X_i \theta_{m[i]})^T K_i^{-1} (y - X_i \theta_{m[i]}) - \frac{N}{2} \log(2\pi)
\]

**step 6: Sorting of the fireflies**

Sort the fireflies in ascending order of their light intensities and find the current best position:

\[
\Omega_{best}^l = \Omega_{[Q]}
\]

**step 7: Movement of the fireflies**

If \( I_i(\Omega_{[i]}) < I_j(\Omega_{[j]}) \), move a firefly \( i \) at position \( \Omega_{[i]} \) toward a brighter firefly \( j \) at position \( \Omega_{[j]} \) by

\[
\Omega_{[i]} = \Omega_{[i]} + \beta_0 \exp(-\gamma r_{ij}^2) / (\Omega_{[j]} - \Omega_{[i]}) + \alpha_l \cdot \text{rand}()
\]

where \( r_{ij} \) is the Euclidean distance between \( \Omega_{[i]} \) and \( \Omega_{[j]} \), \( \beta_0 \) is the attractiveness at \( r_{ij} = 0 \), \( \gamma \) is the media absorption coefficient, \( \alpha_l \) is the randomization parameter, and \( \text{rand}() \) is uniformly distributed random number with amplitude in the range \([-0.5, 0.5] \). \( \beta = \beta_0 \exp(-\gamma r_{ij}^2) \) is the attractiveness between the fireflies \( i \) and \( j \).

**step 8: Repetition**

Set the iteration counter to \( l = l + 1 \) and go to step 2 until the prespecified iteration number \( l_{max} \).

Finally, at the termination of this algorithm when \( l = l_{max} \), the suboptimal \( \hat{\Omega} = [\hat{\theta}_c^T, \hat{\theta}_p^T]^T \) and the corresponding \( \hat{\theta}_m \) are determined by the best position \( \Omega_{best}^{l_{max}} \) of firefly.

**VI. Prediction by GP Posterior Distribution**

Let the amount of electric power damage corresponding to the estimated typhoon weather data \( x_* = [x_{1*}, x_{2*}]^T \) in the Amami archipelago be \( y_* \). Then, we can get the joint Gaussian distribution of \( y \) and \( y_* \) under the GP prior as

\[
\begin{bmatrix} y \\ y_* \end{bmatrix} \sim \mathcal{N} \left( \begin{bmatrix} m(X) \\ m(x_*) \end{bmatrix}, \begin{bmatrix} K & \Sigma(X, x_*) \\ \Sigma(x_*, X) & s(x_*, x_*) + \sigma_n^2 \end{bmatrix} \right)
\]

where \( \Sigma(X, x_*) = \Sigma^T(x_*, X) \) is the \( N \)-dimensional covariance vector evaluated at all pairs of the training input \( X \) and the new input \( x_* \). From the formula for conditioning a joint Gaussian distribution, the posterior distribution for \( y_* \) is obtained as

\[
y_* \mid X, y, x_* \sim \mathcal{N}(\hat{y}_*, \hat{\sigma}_n^2)
\]

where \( \hat{y}_* \) is the predictive mean and \( \hat{\sigma}_n^2 \) is the predictive variance, which are given as follows:

\[
\hat{y}_* = m(x_*) + \Sigma(x_*, X)K^{-1}(y - m(X))
\]

\[
\hat{\sigma}_n^2 = s(x_*, x_*) - \Sigma(x_*, X)K^{-1}\Sigma(X, x_*) + \sigma_n^2
\]

\( \hat{y}_* \) is the predicted amount of electric power damage by the typhoon and \( \hat{\sigma}_n^2 \) is utilized as the confidence measure of the predicted amount of damage.

**VII. Simulations**

We predict the amount of electric power damage using the actual data of 18 typhoons that hit or came close to the Amami archipelago from 1996 to 2009. These 18 typhoon data are divided into 17 typhoon data for training and 1 typhoon datum for prediction. Namely, we can predict the electric power damage with 18 combinations of training and prediction data. The amount of electric power damage is taken to be the number of the power failure circuits. The setting parameters of FA are chosen as follows:

(i) firefly size: \( Q = 100 \)
(ii) attractiveness at \( r_{ij} = 0 \): \( \beta_0 = 1.0 \)
(iii) media absorption coefficient: \( \gamma = 1.0 \)
(iv) randomization parameter: \( \alpha_l = 1.0 \times (0.97)^l \)
(v) maximum iteration number: \( l_{max} = 100 \)

The prediction result obtained by the proposed method is shown in Fig. 2. In this figure, the circles show the true number of the power failure circuits, the squares show the predicted number of the power failure circuits, and the shaded areas give the double standard deviation confidence interval (95.5% confidence region). The actual damages of 15 typhoons are included in the double standard deviation confidence interval. The probability that the actual damages are included in the double standard deviation confidence interval is 83.3%. This indicates that the proposed method yields quite reasonable confidence region of the predicted amount of damage.

For comparison, the two-stage prediction method [5] is applied to this prediction problem. The prediction result obtained by the two-stage prediction method is shown in Fig. 3. The average error rate:

\[
E = \frac{1}{18} \sum_{k=1}^{18} \frac{|y_*(k) - \hat{y}_*(k)|}{y_*(k)}
\]

is calculated for the proposed method and the two-stage prediction method, where \( y_*(k) \) is the actual damage, i.e., the true number of the power failure circuits, and \( \hat{y}_*(k) \) is the predicted number of the power failure circuits. As a result, the average error rate is 0.483 for the proposed method and 0.665 for the two-stage prediction method. The average error rate of the proposed method is 27.4% smaller than that of the two-stage prediction method. Therefore, we conclude that the accuracy of the propose method is superior to that of the conventional two-stage prediction method.

It should be noted that any confidence measures of the predicted amount of damage could not be obtained in the
two-stage prediction method. On the other hand, the proposed method can give not only the predicted values but also the confidence regions of the predicted values. Therefore, in effect, we can utilize the upper value of the confidence region \( \hat{y}_{\text{max}} = \hat{y} + 2\sigma \) as the predicted value of the worst case. This suggests that the proposed method can reduce the possibility that the staff and materials necessary for restoration are lacking in the Amami archipelago. This is also one of the advantages of the proposed method.

VIII. CONCLUSIONS

In this paper, an application of FA to GP-based prediction of electric power damage caused by typhoons has been presented. The separable LS approach combining the linear LS method with FA has been successfully applied to training of the GP prior model. The hyperparameter vector of the GP prior covariance and the adjusting parameter vector of the typhoon track are searched for by FA, while the weighting parameter vector of the GP prior mean is estimated by the linear LS method. Since FA is simple and has a high potential for global optimization, the proposed training algorithm is efficient for construction of the prediction system. Simulation results show that the proposed prediction system yields accurate predicted amount of damage and reasonable confidence region. Comparison with other population-based algorithms is one of the future works.

ACKNOWLEDGMENT

The authors would like to express sincere thanks to Kyushu Electric Power Company, Kagoshima Branch, for offering their data and support in this research.

REFERENCES