The Ability of Dual-baseline Interferometer to Resolve Angular Ambiguities

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Abstract—A rigorous mathematical analysis of the ability of dual-baseline Interferometer to resolve angular ambiguities based on Chinese remainder theorem is presented. Theoretical analysis and simulation results show that by properly designing the antenna spacing, it is possible to improve significantly the ability to resolve angular ambiguities under the constraint that the distances between adjacent antenna elements are relatively prime.

Keywords—ambiguity number, dual-baseline interferometer, resolving phase ambiguity, remainder theorem.

I. INTRODUCTION

PHASE interferometer with high measurement accuracy is the field of electronic reconnaissance generally adopted a direction finding method. For phase unwrapping, interferometer direction finding system usually adopts multiple baseline system [1-6]. Literature [2] points out that the dual baseline system based on remainder set comprehension phase fuzzy capacity limited to the maximum phase error and two of baseline length corresponding to the ratio of the two prime numbers. The [3] phase unwrapping sufficient conditions are: long, short baseline maximum direction finding error and "should be less than long baseline minimum half a fuzzy zone", and "the short baseline direction finding fuzzy zone number less than long baseline array fuzzy zone number, the number difference of at least 1". The theory analysis and simulation results show that the [2], [3] conclusion is inaccurate. Then, fuzzy solution ability restriction factor is what? Which be discussed in this paper is the core problem.1

1. Based on the understanding of the principle of phase remainder theorems of Fuzzy

Fig 1 is a dual baseline interferometer based on remainder theorems of fuzzy understanding of phase diagram. In the picture, Antenna unit 1 and 2, 2 and 3 respectively form a baseline 1, 2. The corresponding baseline length and phase discriminator error output respectively $d_{12}$, $d_{23}$ ( $d_{12}/d_{23} = p/q$, p, q is prime number), $\phi_{12}$, $\phi_{23}$ (When considering the phase error, Set the $\phi_{12}$, $\phi_{23}$ were written as $\hat{\phi}_{12}$, $\hat{\phi}_{23}$); $\theta$, $\lambda$ are derived from the far-field electromagnetic signal arrival angle and wavelength of a signal. Fig 1 available :

![Image](image.png)

Fig.1 Double baseline interferometer phase unwrapping principle diagram

$$\phi_{12} + 2k_{01}\pi = \frac{2\pi d_{12}}{\lambda} \sin \theta$$

$$\phi_{23} + 2k_{02}\pi = \frac{2\pi d_{23}}{\lambda} \sin \theta$$

$$d_{23}(\phi_{12} + 2k_{01}\pi) - d_{12}(\phi_{23} + 2k_{02}\pi) = 0$$

In which, $k_{01}$, $k_{02}$ respectively corresponds to the baseline 1, 2 phase fuzzy value. Based on the type (4) in $k_1$, $k_2$ twodimensional search method for finding a minimum of $|\Delta_{err}|$

The minimum value of the corresponding $k_{01}$, $k_{02}$ will correspond to the true phase fuzzy values $k_{01}$, $k_{02}$. This is the double baseline interferometer based on remainder set understanding phase and fuzzy theory.

$$\Delta_{err} = d_{23}(\phi_{12} + 2k_1\pi) - d_{12}(\phi_{23} + 2k_2\pi)$$

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2. Solution of fuzzy analysis

2.1 Remainder theorems of fuzzy principle understanding another expression

If Fig 1 phase discriminator 1, 2 phase errors were \( \Delta \phi_{21}, \Delta \phi_{23} \), so (1), (2) can be expressed as a formula (5), (6) the form of display

\[
\phi_{21} + \Delta \phi_{21} + 2k_0 \pi = \hat{\phi}_{21} + 2k_1 \pi = \frac{2 \pi d_{21}}{\lambda} \sin \theta_{k_1} \\
\phi_{23} + \Delta \phi_{23} + 2k_0 \pi = \hat{\phi}_{23} + 2k_2 \pi = \frac{2 \pi d_{23}}{\lambda} \sin \theta_{k_2}
\]

(5)

(6)

In which, \( \hat{\phi}_{k_1} - \theta, \hat{\phi}_{k_2} - \theta \) respectively are indicated by \( \Delta \phi_{21}, \Delta \phi_{23} \) error \( \theta_{k_1}, \theta_{k_2} \) respectively express with or without the phase error, phase fuzzy values \( k \theta \) the estimated value. When the \( k_1, k_2 \) two-dimensional search, search value and true value that the difference is exist, they are equal to \( \theta \) (This is the remainder theorems of understanding phase and fuzzy theory) ; 2) When \( \theta_{k_1} \neq \theta_{k_2} \), \( \min \left( \left| \theta_{k_1} - \theta_{k_2} \right| \right) \) (min(●) said minimum). The corresponding two fuzzy direction of arrival with different a values.

In order to understand based on remainder set comprehension phase fuzzy principle. Fig 2 show when \( \lambda \), \( d_{21} \) and \( d_{23} \) are constant, the value of two \( \theta \) cases \( \theta_{k_1}, \theta_{k_2} \) location interrelation between (Diagram of the units of length are meters, visual display, \( \theta_{k_1} \) and \( \theta_{k_2} \) are calculated 100 times to form a straight line) . Fig 2 shows: 1) In the \( \theta_{k_1} \) and \( \theta_{k_2} \) of all possible values of a set of values, only exist, they are equal to \( \theta \) (This is the remainder theorems of understanding phase and fuzzy theory) ; 2) When \( \theta_{k_1} \neq \theta_{k_2} \), \( \min \left( \left| \theta_{k_1} - \theta_{k_2} \right| \right) \) (min(●) said minimum). The corresponding two fuzzy direction of arrival with different a values.

Obviously, when no phase error, based on remainder set comprehension phase fuzzy method is effective. When the existence of phase error, \( \left| \hat{\phi}_{k_1} - \theta \right| \) is likely to be \( \min \left( \left| \theta_{k_1} - \theta_{k_2} \right| \right) \) \( \left( \theta_{k_1} \neq \theta_{k_2} \right) \) corresponding to the fuzzy direction near the minimum value, which causes errors in ambiguity. In other words, type (8) below condition does not always hold

\[
\Delta \theta_{k_1,k_2} = \min \left( \left| \theta_{k_1} - \theta_{k_2} \right| \right) \left( \theta_{k_1} \neq \theta_{k_2} \right)
\]

2.2 Solution of fuzzy analysis

Hypothesis in the entire angular range corresponding to the baseline \( d_{21}, d_{23} \) maximum angle error are \( \Delta \theta_{1_{\text{max}}} \) and \( \Delta \theta_{2_{\text{max}}} \). In Fig 3, \( k_1 \) and \( k_2 \) are phases fuzzy value true value; \( \Delta k_1, \Delta k_2 \) respectively are \( k_1, k_2 \) estimation error, all integer, \( \left| \Delta k_1 \right| + \left| \Delta k_2 \right| \neq 0 \), \( \theta_0 \) is signal DOA ; \( \Delta \theta_{k_1,k_2} = \min \left( \left| \theta_{k_1} + \Delta k_1 - \theta_{k_2} + \Delta k_2 \right| \right) \)

Fig 3 shows :

\[
\left| \hat{\theta}_{k_1} - \theta_{k_1} \right| \leq \Delta \theta_{1_{\text{max}}} + \Delta \theta_{2_{\text{max}}} \Rightarrow \text{only when the formula}
\]

Fig 3 The remainder must understand fuzzy capability analysis diagram
below the condition is satisfied, in order to correct the phase ambiguity.

\[ |\hat{\theta}_{2k+\Delta k} - \theta_{k+\Delta k}| > \Delta \theta_{1\text{max}} + \Delta \theta_{2\text{max}} \]  \hspace{1cm} (9)

so

\[ \Delta \theta_{1\text{max}} + \Delta \theta_{2\text{max}} < \frac{1}{2} \Delta \theta_{k_1 k_2\text{min}} \]  \hspace{1cm} (10)

As you can see from Fig 3, \( \Delta \theta_{k_1 k_2\text{min}} \) usually significantly smaller than the long baseline half a fuzzy zone, therefore, [3] literature to fuzzy solution conditions too loose. From the type (10) visible, two baseline phase error is the solution of fuzzy factors in the literature, so the [2] conclusion is naturally less accurate.

3. The correct phase unwrapping of phase error request

According to the interferometer angle measurement error formula

\[ \Delta \theta = \left( \frac{\Delta \lambda}{\lambda} - \frac{\Delta d}{d} \right) \tan \theta + \frac{\lambda |\Delta \phi|}{2 \pi d \cos \theta} \]  \hspace{1cm} (11)

And consider the worst case, available \( (\Delta \lambda/\lambda , \Delta d/d) \) separately for the signal wavelength to estimate the relative error and the relative error of the baseline installation.

\[ \Delta \theta_{\text{max}} = \left( \frac{\Delta \lambda}{\lambda} + \frac{\Delta d}{d} \right) \tan \theta + \frac{\lambda |\Delta \phi|}{2 \pi d \cos \theta} \]  \hspace{1cm} (12)

\[ \Delta \theta_{\text{1max}} = \left( \frac{\Delta \lambda}{\lambda} + \frac{\Delta d_{12}}{d_{12}} \right) \tan \theta + \frac{\lambda |\Delta \phi_{12}|}{2 \pi d_{12} \cos \theta} \]  \hspace{1cm} (13)

\[ \Delta \theta_{\text{2max}} = \left( \frac{\Delta \lambda}{\lambda} + \frac{\Delta d_{23}}{d_{23}} \right) \tan \theta + \frac{\lambda |\Delta \phi_{23}|}{2 \pi d_{23} \cos \theta} \]  \hspace{1cm} (14)

\[ d_{12} |\Delta \phi_{12}| + d_{23} |\Delta \phi_{23}| < \]  \hspace{1cm} (15)

\[ d_{12} d_{23} \tan \theta \left[ \frac{\Delta \theta_{k_1 k_2\text{min}}}{\lambda} - 2 \left( \frac{\Delta \lambda}{\lambda} + \frac{\Delta d_{12}}{d_{12}} + \frac{\Delta d_{23}}{d_{23}} \right) \tan \theta \right] \]  \hspace{1cm} (16)

By formula (16) visible, \( d_{12} + d_{23} \) is a constant value, The correct phase unwrapping required phase error is not only related to the product of \( d_{12} \text{ and } d_{23} \), Also with \( \Delta \theta_{k_1 k_2\text{min}} , \theta, \lambda, \Delta \lambda/\lambda, \Delta d_{12}/d_{12} \text{ and } \Delta d_{23}/d_{23} \) related.
Fig. 4 $d_{12}$ and $d_{23}$ different value combinations of circumstances,

The relation curve between $\frac{d_{12}d_{23}\Delta \theta_{k_i_{\min}}}{\lambda}$ and $\lambda$

<table>
<thead>
<tr>
<th>Table I</th>
<th>$d_{12}$ · $d_{23}$ SEVERAL VALUES COMBINATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_{12}$</td>
<td>0.39</td>
</tr>
<tr>
<td>$d_{23}$</td>
<td>0.08</td>
</tr>
</tbody>
</table>

II. CONCLUSION

Through strict mathematical derivation, the double baseline interferometer based on remainder set comprehension phase ambiguity, the correct phase unwrapping required phase error and two of baseline length, angle of arrival of two baseline fuzzy difference minimum, angle of arrival, the signal wavelength, the signal wavelength to estimate the relative error and the relative error between the baseline installation magnitude relation. Preliminary simulation results show that, in the baseline length meet the remainder theorem condition, still can use the baseline length reasonable design to improve the solution of fuzzy ability.

REFERENCES

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