Abstract—we consider a joint inventory-location problem with two classes of fast and slow moving products for after sales services network. Suppose an after sales services network with a plant and multiple dealers to serve the customers in markets. We assume the location of plant and dealers are given and the problem is going to locate some warehouses to stock the slow-moving products and then assigning dealers to these warehouses in order to call the demand of slow-moving products. Other decisions that are made in this problem is assigning the markets to dealers and obtaining the inventory levels of slow-moving in warehouses and fast-moving in dealers. The derived model is a mixed non-linear integer programming model and a simulated annealing algorithm is proposed to solve it. Finally the computational results and analysis are presented.

Keywords—location/ allocation, inventory, Simulated annealing.

I. INTRODUCTION

Suppose a car manufacture system that needs a network of dealers to serve the after sales services to whom have a car produced by this company. When a car comes to a dealer for service, it needs a range of spare parts to complete the service. Therefore each dealer should stock a suitable inventory of spare parts to serve the customers. The spare parts can be categorized in two classes of fast moving and slow moving parts. It is trivial that each dealer should maintain fast moving parts in its inventory in order to serve satisfactorily the customers. However an important question is, should each dealer maintain the slow moving parts or it is optimal to have some warehouses among the dealers and slow moving parts just are delivered from plant to warehouses and warehouses deliver the need of slow moving parts of other dealers.

The problem is going to determine the number of warehouses needed and locate them among the dealers. In addition the markets assigns to dealers to call their services and the inventory level of fast moving parts in each dealer and the inventory level of slow moving parts in each warehouse are obtained.

This model is applicable for any company that produces a product and needs a network of dealers to serve the after sales services.

II. LITERATURE REVIEW

The traditional inventory control researches focus on determining inventory policies in order to minimize the inventory-related costs (see for example Nahmias 1997 and Zipkin 1997 for a review [1],[3]). Most of the papers in this area assumed a given distribution structure, with given DC locations and known customers assignment to DCs. On the other hand, the facility location literature focus on the facility location considering product transportation costs and usually ignoring or simplifying inventory-related costs (see, for example, Daskin 1995, Mirchandani and Francis 1990, Dreznner 1995, and Geoffrion and Power 1995 for a review [4]-[7]).

Researches on joint inventory location problem began in 1957 by Baumol and wolf [8]. Eppen (1979) showed the cost of centralized system is lower than a decentralized one [9]. Barahona and Jensen (1997) considered an integrated location inventory model and approximated safety stock as a linear function of number of DCs [20]. Erlebacher and Meller (2000) formulate a joint inventory location model with a nonlinear objective function [10]. They used continuous approximation and two heuristics to determine the number of DCs and locating them and assigning customers to DCs. Nozick and Turnquist (2001) considered a production distribution system to determine the number and location of DCs [11]. They used two heuristics to determine inventory levels in DCs as well as the location of DCs. Daskin, et al., (2002) and Shen, et al.,(2003) formulated a joint inventory location model with risk pooling in order to determine the number and location of DCs as well as shipment sizes, the

Qi, et al (2010) studied the effects of facility disruptions at two supply chain echelons to optimize the location of retailers and assignment of customers to retailers while the supply disruptions may occur at either the supplier of the retailers. Chen Q, et al (2011) proposed a joint inventory location model for optimal facility location and inventory management under facility disruptions [19]. The model allows customer re-assignments and minimizes the expected total system cost across all facility disruptions. All of the location-inventory models mentioned above do not consider separate warehouses for categorized products. In this article we incorporate this concept.

III. MODEL FORMULATION

We consider a distribution system with one plant and multiple dealers, warehouses and markets. We assume that the location of plant, dealers and markets are known and infinite storage capacity for dealers and warehouses. We also assume the plant has infinite production capacity. The dealers order inventory from the plant using an economic order quantity model (EOQ). The frequency of orders and order quantity at each dealer is determined by the mean demand served by dealers which depends to the assignment of markets to dealers. In addition, the order quantity and the frequency of orders at each warehouse is a function of assignments of dealers to warehouses. We assume the same unit cost for shipping the fast moving parts from plant to dealers and identical unit cost of shipping slow moving parts between warehouses and dealers. In the following the steps of formulating the problem are presented.

A. Indices

The indices used to formulate the problem are:

- $l$: index for markets, $l=1,...,L$.
- $k$: index for dealers, $k=1,...,N$.
- $r$: index for warehouses, $r=1,...,N$.
- $i$: index for fast moving parts, $i=1,...,I$.
- $j$: index for slow moving parts, $j=1,...,J$.

B. Notation

The parameters ate as follows:

- $P_f, P_s, P_k$: The probability that a customer requires a fast moving part, slow moving part or both of them.
- $q_{ij}$: The probability that a customer needs the fast moving part $i$.
- $q_{2j}$: The probability that a customer needs the slow moving part $j$.
- $q_{3ij}$: The probability that a customer needs both fast moving part $i$ and slow moving part $j$.
- $c_{ik}$: Unit cost of shipping fast moving parts from plant to dealer $k$.
- $c_{2r}$: Unit cost of shipping slow moving parts from plant to warehouse $r$.
- $f_r$: fixed cost of opening a warehouse at dealer $r$.
- $\varepsilon_{1k}$: The lead time of delivering fast moving parts from plant to dealer $k$.
- $\varepsilon_{2r}$: The lead time of delivering slow moving parts from plant to warehouse $r$.
- $A_1, A_2$: the fixed ordering cost of fast moving/ slow moving parts from plant to dealer/ warehouse.
- $\mu_l$: The annual mean demand of customers for market $l$.
- $\sigma_i^2$: Variance (daily) of customers for market $l$.
- $\alpha$: Service level.
- $Z_a$: Standard normal value with respect to $p(z \leq z_\alpha) \leq \alpha$.
- $d_{ki}, d_{2ij}$: the unit cost of shipping a fast moving/ slow moving parts from dealer $k$ to market $l$.
- $h_1, h_2$: Inventory holding cost per unit of fast moving/ slow moving product per year.
- $e_{rk}$: Unit cost of shipping a slow moving part from warehouse $r$ to dealer $k$.
- $\varphi_{rk}$: The lead time of delivering slow moving parts from warehouse $r$ to dealer $k$.
- $P$: Penalty cost of waiting time in dealers to receive slow moving parts.

The decision variables needed to formulate the problem are as follows:

- $X_{kl}$: A zero- one variable for covering of markets by dealers; 1 if dealer $k$ serves market $l$; 0 otherwise.
The objective function minimizes:

- The cost of shipping fast moving products from plant to dealers and from dealers to markets
- The ordering cost and holding cost for fast moving products at dealers.
- The safety stock cost for fast moving products at dealers and the safety stock cost for slow moving products at warehouses.
- The fixed cost of locating warehouses.
- The cost of shipping slow moving products from plant to warehouses and from warehouses to dealers and the penalty cost.
- The ordering cost and holding cost for slow moving products at warehouses.

The first constraint ensures that each market assigns to exactly one dealer. The second constraint stipulates that each dealer assigns only to one warehouse. The third constraint, states that dealers can be assigned to candidate sites that are selected as warehouses. The rest of constraints are no negativity and internality constraints.

**Analysing the model**

In this sub-section we analyze the structure of the model to find the optimal inventory level and properties of optimal solution.

**Lemma 1:**

Suppose the assignment of markets to dealers and the assignment of dealers to warehouses are given. Then the objective function is convex in inventory level and the optimal inventory levels are:

\[ Q_{ki}^* = \frac{2A_1 D_{ki}}{h_1} \quad \text{And} \quad Q_{rj}^* = \frac{2A_2 D_{rj}}{h_2} \]

**Proof:** By first and second derivative conditions we have:

\[ \frac{\partial z}{\partial Q_{ki}} = -A_1 \frac{D_{ki}}{Q_{ki}^2} + \frac{h_1}{2} \]

\[ \frac{\partial^2 z}{\partial Q_{ki}^2} = 2A_1 \frac{D_{ki}^2}{Q_{ki}^3} > 0 \]

Therefore the objective function is convex in \( Q_{ki} \) and the optimal inventory levels can be determined as:

\[ Q_{ki}^* = \sqrt{\frac{2A_1 D_{ki}}{h_1}} \]

With the same approach for \( Q_{rj} \), we have:

\[ Q_{rj}^* = \sqrt{\frac{2A_2 D_{rj}}{h_2}} \]

Therefore for any set of locations and allocations the optimal inventory levels in dealers and warehouses are...
calculated by lemma 1. Now we can substitute the optimal equations of $Q_{ki}$ and $Q_{rj}$ in the objective function and by relaxing it in these variables the model transforms to an integer programming model as follows:

$$
\min z = \sum_{k} \sum_{i} \sum_{l} M_i \mu_i (c_{ik} + d_{ikl}) X_{kl} + \\
\sum_{k} \sum_{i} \sqrt{2A_i h_i \sum_{l} \mu_i X_{kl} M_i} + \\
h_1 z_{\alpha} \sum_{i} \sum_{l} \sqrt{\xi_2 \sum_{j} \mu_j^2 \Sigma_j X_{kl} + \sum_{r} f_r W_r} + \\
\sum_{r} \sum_{k} \sum_{l} \sum_{j} (c_{2r} + e_{rk} + p \xi_{rk}) \mu_j N_j X_{kl} Y_{rk} + \\
\sum_{k} \sum_{l} \sum_{j} \mu_j N_j d_{2kl} X_{kl} + \\
\sum_{r} \sum_{l} h_2 z_{\alpha} \sum_{j} \sum_{k} \sum_{l} N_j^2 \sigma_i^2 X_{kl} Y_{rk}
$$

Subject to:

$$
\sum_{k} X_{kl} = 1, \forall l
$$

$$
\sum_{r} Y_{rk} = 1, \forall k
$$

$$
Y_{rk} \leq W_r, \forall r, k
$$

$$
X_{kl} \in \{0,1\}, Y_{rk} \in \{0,1\}, W_r \in \{0,1\}
$$

Where

$$
M_i = p_f q_{li} + p_f \sum_{j} q_{3ij}
$$

$$
N_j = p_s q_{2j} + p_f \sum_{i} q_{3ij}
$$

**IV. SOLUTION APPROACH**

The proposed model is categorized into integer nonlinear programming problems which are well-known to be NP-hard; because they are generalization of mixed integer linear programming problems which themselves are NP-hard. Therefore it is fully justified to employ heuristic and meta-heuristic algorithms, including simulated annealing, to efficiently solve this problem in a reasonable amount of time. To do so, we propose a simulated annealing (SA) algorithm. SA is a random search technique that has been used to solve many combinatorial optimization problems.

In order to solve the problem under consideration three types of decision should be made: assignment of markets to the dealers, grouping of the dealers and determining the warehouse in each group. For the first two decisions, two SA algorithms are used and the third decision is made through enumeration. In the following sections different elements of the proposed SA is presented.

**4.1. Initial solution and neighborhood structure**

The initial solution is generated randomly; this solution determines assignment of markets to the dealers. In other words each market is assigned to a randomly selected dealer. In order to generate a neighborhood solution market is randomly selected from a non-empty dealer and assigned to another randomly selected dealer.

**4.2. Solution evaluation**

In order to evaluate a solution i.e. compute the costs, grouping of the dealers and the warehouse in each group must be determined, these decisions are made using another SA procedure. In other words another SA procedure is used to estimate optimum grouping of dealers, subject to the given assignment of markets to the dealers. This procedure is called ISA (Inner simulated annealing) and explained in the following subsections.

**4.2.1. Initial solution and neighborhood structure for ISA**

Initial grouping of the dealers is generated randomly, and a neighborhood solution is generated by randomly selecting a dealer and assigning it to another group.

**4.2.2. Solution evaluation and cooling schedule for ISA**

So far the assignment of markets to the dealers and the grouping of the dealers are determined and in order to be able to compute the corresponding costs, for each group of dealers, one dealer must be selected as a warehouse, in this paper all of the possible selections of warehouses are enumerated and the best selection of the warehouses is determined. In order to control the search mechanism, determining the trend in which the temperature drops is of major importance. A proper trend of reducing the temperature prevents the algorithm from biasing toward bad neighbors throughout the search. In this paper an exponential cooling schedule is employed. This cooling schedule updates the temperature using the following formula:

$$
T_i = \frac{A}{i+1} + B, i = 1, ..., n
$$

In this formula, $A = \frac{(T_0 - T_f)(n+1)}{n}, B = T_0 - A$

$n$ is the number of temperature updates, $T_0$ is the beginning temperature and $T_f$ is the final temperature. In this study the number of neighborhood search in each temperature is considered to be 50. In other words at each temperature 50 neighbors are examined. The initial temperature and the number of temperatures to be considered are 125 and 300 respectively. These values are obtained after several experiments on different instances.
4.2.3. Termination criterion for ISA
In this paper a maximum number of temperature updates, equal to 300, is considered as the termination criterion for the ISA.

4.3. Cooling schedule and termination criteria
In this paper an exponential cooling schedule is used for the main SA procedure. The details of this type of cooling schedule are presented in section 1.2.2. Reaching the final temperature, $T_f$, or exceeding a predetermined computational time is considered as the termination criteria. After performing several experiments on different instances the number of neighborhood search in each temperature is set to be 50 also $T_0$ and $T_f$ for the main SA procedure are set to be 125 and 0.1 respectively. Finally the maximum running time is set to be 100*N*L milliseconds; Where N and L are respectively the number of dealers and the number of markets.

Figure 1 shows the pseudo code for the main SA procedure, as seen from this figure, the procedure begins with a random initial solution then it is evaluated using the ISA procedure. Afterwards using the neighborhood generation structure explained in previous sections a new solution, NX, is generated and evaluated using the ISA procedure, if the corresponding cost for NX is less than that of the current solution, it is accepted as the new current solution otherwise it is accepted with the probability of $e^{(-f_{T_f} - f_{T_{bestNX}})}$, this process is repeated for a certain number of iterations. Then the temperature is updated using the exponential cooling procedure.

5. OPTIMAL POLICY IN DIFFERENT CONDITIONS
We tend to know in which conditions it is better that we have a number of warehouses and in which conditions it is more effective to construct each warehouse in each dealer. So we solved a lot of different problems to discover an optimal policy and the following are their results.

1- When the transportation cost between warehouses and dealers are more significant than maintenance cost of slow moving products it is more effective to have more warehouses; most dealers should maintain their slow moving products especially when their demand is high. Dealers in which the demand of slow moving products is low can be assigned to other warehouses in order to provide their slow moving products.

2- In equal conditions, by increasing the demand of slow moving products, more dealers should serve as warehouses.

3- In equal conditions, increase of demand indicates that it is better that more dealers maintain their slow moving products; when the number of markets increases and as a result the demand of dealers is increasing, it is better that we use each dealer as warehouse.

4- In cases which the satisfactory level of customers is important (high penalty cost) and the transportation cost between dealers and warehouses are not high, it is more effective to locate each warehouse in each dealer.

REFERENCES