Abstract— In this paper, the stability limitations of voltage source converter (VSC) based high-voltage direct-current (HVDC) link analyzed by power-synchronization control for alternating-voltage control mode. The analysis has been done by the two theories i.e., phasor approach and the space-vector approach. The phasor approach is straightforward to apply and gives perceptive results, the space-vector approach reflects more dynamic insights of the system. In state space-vector using Jacobian transfer matrices, dynamic models of VSC and ac system are developed. This approach shows that right half plane (RHP) zeros impose a fundamental limitation on the achievable bandwidth of the VSC-HVDC link. At moderate voltage levels, a VSC-HVDC link operating in alternating-voltage control mode can achieve higher bandwidth because (RHP) zero is located further from the origin.

Keywords— Control, power electronics, power systems, stability.

I. INTRODUCTION

VOLTAGE-SOURCE converter (VSC) based high-voltage direct-current (HVDC) Transmission has gained Importance in recent years [1]-[4]. The VSC-HVDC system can be connected to very weak ac systems [5]. Various control methods have been proposed for VSC-HVDC systems. Power-angle control is simple and straightforward to implement. One disadvantage of power-angle control is that the control bandwidth is limited by a resonant peak at the grid frequency. Vector-current control gives poor performance for VSC-HVDC links connected to weak ac systems. The recently proposed power-synchronization control is particularly suitable for controlling a VSC-HVDC link connected to weak ac systems [6]. VSC-HVDC link is subjected to various operation limitations. Research conducted focused on the limitations of the converter itself, such as converter-current limitation and modulation-index limitation, etc., but not on limitations originating from grid-interaction stability issues.

There are two approaches for such an analysis. One is the phasor approach. And gives simple analytical results. One drawback of this approach cannot give instantaneous values. Space-vector theory is another possible approach. Since the space-vector theory is based on instantaneous-value representation, it consider electromagnetic transients in electrical circuits [7], which have a particular importance for control of high power-electronic devices, such as FACTS and HVDC systems. By making linearizations, space-vector theory is a practical tool for investigation of the stability of VSC-HVDC systems. This paper is a continuation of [6], where the basic principle of power-synchronization control is proposed. It analyzes the stability limitations of ac system on a VSC-HVDC link using the recently proposed power-synchronization control for alternating-voltage control mode. The paper is organized as follows. In Section II, the stability limitation of a normal VSC-HVDC link is described using the phasor approach. In Section III, the space-vector approach is applied to derive the Jacobian transfer matrix for VSC and ac system. By using the results from Section III, stability limitations are analyzed by the Jacobian transfer matrices for a VSC-HVDC link connected to an ac system in Section IV. In Section V, the theoretical analysis presented in the paper is verified by time simulations using the software PSCAD.

Fig. 1 shows the main-circuit diagram of a VSC-HVDC converter connected to an ac system. \( L_s \) and \( R_c \) are the inductance and resistance of the phase reactor, and \( L_g \) and \( R_g \) are the inductance and resistance of the ac system. \( C_f \) is the ac capacitor connected at the point-of-common-coupling (PCC). The bold letter symbols, \( \mathbf{E} \), \( \mathbf{u}_f \) and \( \mathbf{v} \) represent the voltage vectors of the ac source, the PCC, and the VSC, respectively. The ac source is used as the voltage reference, and the phase angles of \( \mathbf{u}_f \) and \( \mathbf{v} \) are \( \theta_u \) and \( \theta_v \), respectively. The quantity \( \mathbf{i}_c \) is the current vector of the phase reactor, and \( \mathbf{i}_g \) is the current vector to the ac source.

II. STABILITY LIMITATIONS OF A VSC-HVDC LINK

The stability limitation of the ac system is analyzed in this section by the phasor approach. To simplify the analysis two assumptions are made:
• The direct voltage is constant due to the assumption that the converter controlling the direct voltage on the other side of the VSC-HVDC link connected to strong ac system
• The ac capacitor \( C_f \) is neglected. This capacitor has a very small value and, the ac capacitor does not have impact on the stability limitation of the VSC-HVDC link [8].
**Alternating-Voltage Control Mode:**

In alternating-voltage control mode, the control system of the VSC keeps the PCC voltage constant and controls the active power to/from the PCC. If the resistance $R_g$ is neglected, the active power $P$ can be expressed by the power-angle equation

$$P = \frac{u_f i_o}{x} \sin \theta_{uo}$$  

(1)

Where, $X = \omega_1 L_g$ and $\omega_1$ is the nominal angular frequency of the ac system. From (1), the power that can be transmitted to/from the converter is limited by the fact that $\sin \theta_{uo} \leq 1$. So, the stability limitation of the VSC in alternating-voltage control mode is the angle-stability limit[7]. For the next two sections, the analysis is made using the space-vector approach.

**III. AC-System Modeling Using Power-Synchronization Control**

The space-vector approach is applied to model the main circuit in Fig. 1. The control system is assumed to use the recently proposed power-synchronization control [6]. The main-circuit models can be written as the following linearized input-output form:

$$\begin{bmatrix} \Delta P \\ \Delta U_f \end{bmatrix} = \begin{bmatrix} J_{P0}(s) & J_{Pv}(s) \\ J_{Uf,0}(s) & J_{Uf,v}(s) \end{bmatrix} \begin{bmatrix} \Delta \theta_v \\ \Delta V \\ J_{P0}(s) & J_{Pv}(s) \end{bmatrix} $$

(2)

Where $\Delta P$, $\Delta U_f$, $\Delta \theta_v$, and $\Delta V$ are the linearized deviation of $P$, $U_f$, $\theta_v$, and $V$. The transfer matrix $J_{P(uo)}$ is the linear model of the main circuit in alternating voltage control. This is named as Jacobian transfer matrices. A fundamental difference between jacobian matrix and jacobian transfer matrices is that the Jacobian transfer matrices describe the input output relationships with the instantaneous-value representation, while jacobian matrix only describes the relationships with the phasor representation. The transfer functions in the jacobian transfer matrix are derived analytically.

**A. Transfer Function $J_{P(\theta)}$:**

In a dq reference frame by neglecting the synchronous ac capacitor $C_f$, the dynamic equation of the main circuit in Fig. 1 can be written in the dq frame:

$$L_{d} \frac{di_d}{dt} = v - E - Ri - j \omega_1 L_i$$  

(3)

Where, $R = R_g + R_c$, $L = L_g + L_c$, and $i = i_o = i_g$. Equation (3) can also be written in component form as

$$L_{d} \frac{di_d}{dt} = V \cos \theta_v - E_o - R I_d + \omega_1 L_i q$$

$$L_{d} \frac{di_d}{dt} = V \sin \theta_v - R I_q + \omega_1 L_i d$$

(4)

Let the voltage magnitude of the VSC be kept constant $V=V_o$. If the operating points in (4) are denoted with subscript "0", and the deviations around the operating points are denoted with the prefix “Δ", (4) can be linearized using

$$\theta_v = \theta_v^o + \Delta \theta_v$$

$$i_q = i_q^o + \Delta i_q$$

(5)

Where $\cos \theta_v$ and $\sin \theta_v$ can be linearized as

$$\cos(\theta_v^o + \Delta \theta_v) \approx \cos \theta_v^o + \cos \theta_v^o \Delta \theta_v$$

$$\sin(\theta_v^o + \Delta \theta_v) \approx \sin \theta_v^o + \cos \theta_v^o \Delta \theta_v$$

(6)

Substituting (5) and (6) into (4), and keeping only the deviation parts yields the linearized form of (4)

$$L_{d} \frac{di_d}{dt} = -V_o \sin \theta_v \Delta \theta_v - R \Delta i_d + \omega_1 L_i q$$

(7)

Applying the Laplace transform to (7),

$$\Delta i_d = \frac{V_o \cos \theta_v}{sL_g + (sL_g + \omega_1 L_i)^2} \sin \theta_v \Delta \theta_v$$

(8)

In space-vector approach, the instantaneous active power $P$ is defined as

$$P = R \{u i^*\}$$

(9)

By linearizing the above equation and represent in component form

$$\Delta P = \begin{bmatrix} \Delta i_d \\ \Delta i_q \end{bmatrix}$$

(10)

The current vector $i_o = i_d + j \omega_L i_q$ at the operation point can be derived by

$$i_o = \frac{U_{fo} \cos \theta_{uo}}{R_g} - E_o + j \omega_1 L_g$$

(11)

From that

$$i_d = \frac{u_f \sin \theta_{uo}}{\omega_1 L_g}$$

(12)

If the resistance $R_g$ neglected the voltage vector of the PCC at the operating point, $u_{uo}=u_{fdo}+u_{fqq}$, can be expressed as

$$u_{fdo} = u_{fo} \cos \theta_{uo}$$

$$u_{fqq} = u_{fo} \sin \theta_{uo}$$

(13)

Linearization of the equation

$$L_{g} \frac{di}{dt} = u_f - E - R_g i - j \omega_1 L_g i$$

(14)
The subdivision of d-q component the above expression rewritten as,

\[ \Delta u_d = sL_d \Delta i_d - \omega_L L_q \Delta i_q \]
\[ \Delta u_q = sL_q \Delta i_q + \omega_L L_d \Delta i_d \]
\[ \Delta P = \frac{a_o s^2 + a_1 s + a_2}{(sL + R)^2 + (\omega_L L)^2} \Delta \theta_v \]

\[ a_o = \frac{L}{\omega_L} (k_3 - k_1) \quad a_1 = \frac{R}{\omega_L} (k_3 - k_1) L k_2 \]
\[ a_2 = \omega_L L k_3 - R k_4 \]  \hspace{1cm} (15)

\[ \Delta U_f = \frac{a_o s^2 + a_1 s + a_2}{(sL + R)^2 + (\omega_L L)^2} \Delta \theta_v \]
\[ \Delta P = \frac{a_o s^2 + a_1 s + a_2}{(sL + R)^2 + (\omega_L L)^2} \Delta V \]
\[ \Delta U_f = \frac{a_o s^2 + a_1 s + a_2}{(sL + R)^2 + (\omega_L L)^2} \Delta V \]  \hspace{1cm} (16)

\[ k_1 = V_o U_f \cos(\theta_v - \theta_u) \quad k_2 = V_o U_f \sin(\theta_v - \theta_u) \]
\[ k_3 = E_o V_o \cos \theta_v \quad k_4 = E_o V_o \sin \theta_v \]  \hspace{1cm} (17)

Similarly in [7], for the transfer functions \( J_{f0}(s) \), \( J_{PV}(s) \), and \( J_{fPV}(s) \)

\[ \frac{\Delta U_f}{J_{f0}(s)} = \frac{a_o s^2 + a_1 s + a_2}{(sL + R)^2 + (\omega_L L)^2} \Delta \theta_v \]
\[ \frac{\Delta P}{J_{PV}(s)} = \frac{a_o s^2 + a_1 s + a_2}{(sL + R)^2 + (\omega_L L)^2} \Delta V \]
\[ \frac{\Delta U_f}{J_{fPV}(s)} = \frac{a_o s^2 + a_1 s + a_2}{(sL + R)^2 + (\omega_L L)^2} \Delta V \]  \hspace{1cm} (18)

Where

\[ a_o = \frac{L}{\omega_L} (E_o V_o \cos \theta_v - V_o^2) \quad a_1 = \frac{L}{\omega_L} (E_o V_o \cos \theta_v - V_o^2) \]
\[ a_2 = \omega_L L E_o V_o \cos \theta_v - R E_o V_o \sin \theta_v \]  \hspace{1cm} (19)

\[ \Delta P = \frac{a_o s^2 + a_1 s + a_2}{(sL + R)^2 + (\omega_L L)^2} \Delta \theta_v \]
\[ \Delta U_f = \frac{a_o s^2 + a_1 s + a_2}{(sL + R)^2 + (\omega_L L)^2} \Delta V \]  \hspace{1cm} (20)

IV. STABILITY LIMITATION IMPOSED BY THE AC SYSTEM

From the transfer functions of jacobian transfer matrices stability limitations on VSC-HVDC link are analyzed for two operation modes.

Alternating-Voltage Control Mode

The VSC keeps the voltage magnitude constant, while the active power \( P \) is controlled by adjusting the phase angle \( \theta_v \). The closed-loop system becomes a single-input-single-output (SISO) feedback control system. \( K(s) \) represents the controller of the VSC. The transfer function \( J_{f0}(s) \) is the linear model of the ac system as defined in Section III. In this case if \( U_{fo} = V_o \) and \( \theta_{uo} = \theta_{vo} \), the transfer function \( J_{f0}(s) \) becomes

\[ \Delta P = \frac{a_o s^2 + a_1 s + a_2}{(sL + R)^2 + (\omega_L L)^2} \Delta \theta_v \]  \hspace{1cm} (21)

\[ \Delta U_f = \frac{a_o s^2 + a_1 s + a_2}{(sL + R)^2 + (\omega_L L)^2} \Delta V \]

Where

\[ a_o = \frac{L}{\omega_L} (E_o V_o \cos \theta_v - V_o^2) \quad a_1 = \frac{L}{\omega_L} (E_o V_o \cos \theta_v - V_o^2) \]
\[ a_2 = \omega_L L E_o V_o \cos \theta_v - R E_o V_o \sin \theta_v \]  \hspace{1cm} (22)

Equation (22) shows that, it has pair of poles.

\[ s = -\frac{R}{L} \pm j \omega_L \]  \hspace{1cm} (23)

These are do not make any problem for traditional components in power systems because of their band width.

In this paper, zeros of \( J_{f0}(s) \) are main interest.
If the resistance R is neglected in (22), \( a_1 = 0 \). Now \( \mathcal{P}_s(s) \) has two symmetrical zeros

\[
\mathcal{P}_s(s) = \pm \sqrt{-\frac{a_2}{a_0}} = \pm \omega_1 \sqrt{-\frac{E_0 \cos \theta_{\nu_0}}{V_0 - E_0 \cos \theta_{\nu_0}}} \tag{25}
\]

The location of the zeros of \( \mathcal{P}_s(s) \) can be divided by the following boarders:

- The border where \( \mathcal{P}_s(s) \) gets zeros at the origin. This is equivalent to
  \[
  E_0 \cos \theta_{\nu_0} = 0 \tag{26}
  \]
  giving
  \[
  \theta_{\nu_0} = \pm 90^\circ.
  \]
- The border where \( \mathcal{P}_s(s) \) gets zeros at infinity. This is equivalent to
  \[
  V_0 - E_0 \cos \theta_{\nu_0} = 0 \tag{27}
  \]
  giving
  \[
  \theta_{\nu_0} = \pm \text{arccos} \left(\frac{V_0}{E_0}\right) \tag{28}
  \]
- The border where \( \mathcal{P}_s(s) \) gets real zeros at \( \pm \omega_1 \). This is equivalent to
  \[
  \frac{E_0 \cos \theta_{\nu_0}}{V_0 - E_0 \cos \theta_{\nu_0}} = 1 \tag{29}
  \]
  giving
  \[
  \theta_{\nu_0} = \pm \text{arccos} \left(\frac{V_0}{2E_0}\right) \tag{30}
  \]

The border gives an idea about how much the zeros limit the achievable bandwidth of the control system, even though it is not a “real” border.

Both the voltage magnitude \( \frac{V_0}{E_0} \) and the phase angle \( \theta_{\nu_0} \) of the VSC affect their locations. With higher phase angle \( \theta_{\nu_0} \), the zeros get closer to the origin border. If the zeros in the right-half plane (RHP), the system is called as non minimum phase system. The RHP zero of system causes an additional time delay, which imposes a fundamental limitation on the achievable bandwidth of the control loop [10]. As shown in Fig. 6, the origin border determined by the phase angle \( \theta_{\nu_0} \), i.e., \( \theta_{\nu_0} = \pm 90^\circ \), while it is independent of the voltage magnitude of the VSC. If the phase angle less than \( \pm 90^\circ \) is dependent on both \( \theta_{\nu_0} \) and \( \frac{V_0}{E_0} \). This important fact is not reflected by the phasor approach.

\[
\text{Fig. 7 Active power step response of the closed-loop system.}
\]

\[
\text{Eo=1.0 p.u Vp=1.0 p.u, } L = 1.0 \text{ p.u, } R = 0.01 \text{ p.u control: kp=100 rad/s, active damping: kv=0.477, } \theta_{\nu_0} = 30^\circ, \text{ dashed } \theta_{\nu_0} = 50^\circ \text{ Dotted: } \theta_{\nu_0} = 70^\circ
\]

\[
\text{Fig.8 Multi-variable feedback control of active power alternating voltage}
\]

And shows that the non-minimum-phase effect becomes more severe with higher phase angles \( \theta_{\nu_0} \). And in fig 8, the VSC-HVDC is supposed to control both the active power and the voltage magnitude at the PCC. The closed-loop system becomes a multi-input-multi-output (MIMO) feedback control system. For a square matrix, the transmission zeros can be obtained simply by its determinant, i.e., the transmission zeros of \( \mathcal{P}(s) \) are the values of s that satisfy

\[
\text{det}[\mathcal{P}(s)] = \mathcal{P}(s)|_{U_{\nu U} = s} - \mathcal{P}(s)|_{U_{\nu V} = s} = 0 \tag{31}
\]

If the resistance is R neglected, and if the transfer functions from Section III are used, the solutions to (28) are given by

\[
\mathcal{P}(s) = \pm \omega_1 \sqrt{-\frac{E_0 \cos \theta_{\nu_0}}{U_{\nu U} - E_0 \cos \theta_{\nu_0}}} \tag{32}
\]

Equation (25) and (32), have the same form, and the difference is that \( U_{\nu U} \) and \( \theta_{\nu_0} \) in (32) replaced by \( V_0 \) and \( \theta_{\nu_0} \) in (25). Due to the fact \( \theta_{\nu_0} > \theta_{\nu_0} \), it means that the controller of the VSC can achieve higher bandwidth by keeping the PCC voltage constant.

\[\text{V. SIMULATION RESULTS}\]

To verify the theoretical analysis, a VSC-HVDC link is built in the time simulation software PSCAD/EMTDC. The simulation setup contains a VSC-HVDC link sending power to a 400-kV ac system. The other end of the VSC-HVDC link is
assumed to be connected to a strong ac system. The parameters of the VSC-HVDC converter are in Table I.

Fig. 9 shows an active-power step response of a VSC-HVDC link operating in alternating-voltage control mode. A 0.05-p.u. active-power step is applied from P=0.9 p.u to P=0.95 p.u at 0.1 s. An IMC multivariable-feedback controller proposed in [9] is applied to decouple the interaction between the active-power control and the alternating-voltage control. The non-minimum-phase phenomenon in the active power plot, i.e., the initial power drop, agrees with the theoretical analysis. Although the operating point is very close to the theoretical limit (θ_t = 90°), the step response is fairly stable.

![Fig. 9. Active-power step response of a VSC-HVDC link operating in alternating-voltage control mode.](image)

<table>
<thead>
<tr>
<th>TABLE I VSC-HVDC SYSTEM PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated apparent power S_{sys}</td>
</tr>
<tr>
<td>Rated (base) voltage U_{sys}</td>
</tr>
<tr>
<td>Maximum valve current I_{max}</td>
</tr>
<tr>
<td>Phase reactor inductance</td>
</tr>
<tr>
<td>Phase reactor resistance</td>
</tr>
<tr>
<td>Converter transformer rating</td>
</tr>
<tr>
<td>Transformer leakage reactance</td>
</tr>
<tr>
<td>Direct voltage V_{dc}</td>
</tr>
<tr>
<td>System frequency f</td>
</tr>
<tr>
<td>Switching frequency f_{sw}</td>
</tr>
</tbody>
</table>

VI. CONCLUSIONS

In this paper, two approaches phasor and state-space-vector are used to analyze the stability limitations of a VSC-HVDC link connected to an ac system.

- The phasor approach is a straightforward to apply and gives perceptive results, the space-vector approach reflects more dynamic insights.
- Right half plane (RHP) zeros impose fundamental limitation on bandwidth of VSC-HVDC link.
- The closed loop system cannot achieve higher band width than the location of RHP zero i.e., tight control at the low frequencies is not possible.
- In alternating-voltage control mode, VSC can operate in higher band width.

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