Effect of Thermal Radiation and Stratification on Natural Convection over an Inclined Wavy Surface in a Nanofluid Saturated Porous Medium

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Abstract—An analysis is carried out to study the effects of thermal stratification and thermal radiation on natural convection in a nanofluid along an inclined wavy surface embedded in a non-Darcy porous medium. A coordinate transformation is employed to transform the complex wavy surface to a smooth surface. Using local similarity and non-similarity method the governing non-dimensional equations are transformed into coupled ordinary non-linear differential equations and then solved using successive linearization method followed by Chebyshev spectral collocation method. The effects of non-Darcy parameter, thermal radiation parameter, thermal stratification parameter, Brownian motion parameter, thermophoresis parameter, amplitude of the wavy surface, angle of inclination of the wavy surface on the heat and nanoparticle mass transfer rates are studied and displayed graphically.

Keywords—Nanofluid, inclined wavy surface, thermal radiation, thermal stratification, non-Darcy porous medium.

I. INTRODUCTION

Nanofluid are the base fluids (such as water, oil, ethylene glycol etc.) prepared by uniform and stable suspension of nanometer-sized particles into them. Nanofluids are used to improve thermal management system in many engineering application such as transportation, micromechanics and instrument, HVAC system and cooling devices, microelectronics, microfluidics, transportation, biomedical, solid-state lighting, manufacturing, high-power X-rays, scientific measurement, material processing, medicine and material synthesis. A detailed review on nanofluids can be found in the book by by Das et al. [1] and collection of literature by Kakac and Pramuanjaroenki [2].

Convective heat and mass transfer in porous media has been widely studied in recent years due to its wide range of engineering applications such as petroleum recovery, filtration processes, solar energy collectors, heat exchangers, geothermal and hydrocarbon recovery. A review of convective heat transfer in porous medium is presented in the book by Nield and Bejan [3] and Ingham and Pop [4]. Several researches have been carried out on natural convection in a fluid saturated porous medium. Thermal stratification on free convection in a fluid saturated porous medium is widely accepted due to its geophysical and industrial applications such as hot dike complexes in volcanic regions for heating of ground water, development of advanced technologies for nuclear waste management, pollutant and contaminant transport in soil etc. Murthy et al. [5] showed that the thermal stratification in nanofluid saturated non-Darcy porous media influence the flow, heat and nanoparticle mass transfer rates. RamReddy et al. [6] discussed the effect of thermal stratification on natural convection over a vertical plate embedded in a nanofluid saturated non-Darcy porous medium. It is shown that the increase in non-Darcy parameter leads to increase in heat transfer rate and decrease in nanoparticle mass transfer rate. Rashad et al. [7] performed a numerical study to investigate the non-Darcy natural convection boundary layer flow along a vertical cylinder embedded in a thermally stratified nanofluid saturated porous medium. Srinivasacharya and Surender [8] presented a non-similar solution to study the effects of thermal and mass stratification on natural convection boundary layer flow over a vertical plate embedded in a porous medium saturated by a nanofluid.

Thermal radiation plays a major role in some industrial applications such as glass production and furnace design, and also in space technology applications, such as comical flight aerodynamics rocket, propulsion systems, plasma physics and space craft reentry aerodynamics which operates at high temperatures, and also in applications involving high temperatures such as nuclear power plant, gas turbines missiles, satellites, space vehicles and aircraft etc. Motsumi and Makinde [9] numerically studied the effect of thermal radiation on a boundary layer flow of nanofluids over a moving flat plate. Hady et al. [10] reported that an increase in the thermal radiation parameter reduces the nanofluid temperature which leads to increase in the heat transfer rate. Kandaswamy et al. [11] studied the effects of thermophoresis and Brownian motion on MHD boundary layer flow of a nanofluid in the presence of thermal stratification due to solar radiation.

The study of heat and mass transfer from the irregular wavy surfaces is of fundamental importance because of its
enhancing heat transfer characteristics. Irregularities in surfaces occur in many practical situations. These irregularities encounter in several heat transfer devices such as microelectronic devices, flat plate solar collectors and flat plate condensers in refrigerators. Siddiqua et al. [12] studied the natural convection flow with surface radiation along vertical wavy surface.

To the best of the authors’ knowledge, it may be noticed that previous studies did not include the effect of thermal stratification and thermal radiation on natural convection over an inclined wavy surface embedded in a non-Darcy porous medium saturated with nanofluid. The present study mainly focused on exploring the effects of thermal stratification, thermal radiation, Brownian motion, thermophoresis, amplitude and angle of inclination of the wavy plate on free convection in non-Darcy porous medium saturated with nanofluid.

II. MATHEMATICAL FORMULATION

Consider the steady laminar incompressible two-dimensional boundary layer free convection flow along a semi-infinite inclined wavy surface embedded in a nanofluid saturated non-Darcy porous medium. The fluid is considered to be a gray, absorbing emitting radiation but non-scattering medium. The wavy plate is inclined at an angle $A$ (0° ≤ $A$ ≤ 90°) to the horizontal. The inclination angle is 0° (for horizontal plate), 90° (for vertical plate) and 0° ≤ $A$ ≤ 90° (for inclined plate). The coordinate system is shown in Fig. 1. The wavy surface is described by $y = \delta(x) = a \sin(\pi x/l)$, where $a$ is the amplitude of the wavy surface, and $l$ is the characteristic length of the wavy surface. The wavy surface is held at constant temperature $T_w$ and constant nanoparticle volume fraction $\phi_v$ and the ambient medium is assumed to be linearly stratified with respect to the temperature in the form $T_\alpha(x) = T_{\alpha,0} + Bx$ where $B$ is a constant varied to alter the intensity of stratification in the medium. The values of $T_w$ and $\phi_v$ are assumed to be greater than the porous medium temperature $T_{\alpha,0}$ and nanoparticle volume fraction $\phi_v$ sufficiently far from the wavy surface.

Fig. 1 Physical model

The porous medium is considered to be homogeneous and isotropic and is saturated with a fluid which is in local thermodynamic equilibrium with the solid matrix. The fluid has constant properties except the density in the buoyancy term of the balance of momentum equation. The governing equations for this problem under the laminar boundary layer flow assumptions, Boussinesq approximation and using the non-Darcy’s flow through a homogeneous porous medium near the inclined wavy surface are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\left(1 + \frac{K_f}{\nu} \sqrt{u^2 + v^2}\right) \left(\frac{\partial u}{\partial y} + \frac{K_f}{\nu} \sqrt{u^2 + v^2} \frac{\partial v}{\partial x}\right) = -\frac{K_f}{\mu} \left(\frac{\partial T}{\partial y}\right) + \frac{K_f}{\mu} \left(\frac{\partial T}{\partial x}\right)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \gamma D_p \left(\frac{\partial \phi_v}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial \phi_v}{\partial y} \frac{\partial T}{\partial y}\right)$$

where $u$ and $v$ are the velocity components in the $x$ and $y$ directions, respectively, $T$ is the temperature, $\phi_v$ is the nanoparticle concentration, $g$ is the acceleration due to gravity, $K$ is the permeability, $\rho_p$ is the density of the base fluid, $\rho_p$ is the density of the particles, $C_p$ is the heat capacitance of the nanoparticles, $\nu$ is the kinematic viscosity of the fluid. $\alpha$ is the effective thermal diffusivity, $\beta$ is the volume expansion coefficient of the nanofluid, $\mu$ is the dynamic viscosity of the fluid, $q_i$ is the radiative heat flux, $D_p$ and $D_T$ are the Brownian diffusion coefficient and the thermophoresis diffusion coefficient $\gamma$ is the ratio between the effective heat capacity of the nanoparticle material and heat capacity of the fluid and $K_f$ is a material parameter which is a measure of inertia impedance of the matrix to account for non-Darcian inertial effects, $K_s$ the mean absorption coefficient and $\sigma$ is the Stefan-Boltzmann constant. The above equations are written on the assumption that the nanosized particles are suspended in uniform distribution in a base fluid to form a nanofluid. When the nanofluid passes through porous media, the suspension of the nanoparticles is maintained using a surfactant or some surface charge technology to prevent their agglomeration and to avoid being captured by the porous matrix.

The boundary conditions are

$$v = 0, T = T_w, \phi = \phi_v$$

$$u = 0, T \rightarrow T_{\alpha,0}, \phi \rightarrow \phi_v \text{ as } y \rightarrow \infty$$

Introducing the stream function $\psi$, transferring the effect of wavy surface from the boundary conditions into the governing equations by the coordinate transformation

$$\xi = x/l, \theta(\xi, \eta) = \frac{T - T_{\alpha,0}}{T_w - T_{\alpha,0}}, \xi(\eta, \xi) = \frac{\phi - \phi_v}{\phi_v - \phi_v}$$

$$\eta = \sqrt{\frac{\gamma}{2}} \frac{R\alpha^{1/2}}{\xi^{1/2}(1 + \delta^2)}$$

and letting $R\alpha \rightarrow \infty$ (i.e., boundary layer approximation), we obtain the following system of non-dimensional equations:
\[ f^* + \frac{2Gr}{\sqrt{1 + \delta^2}} = f^* f^* = (\sin A + \delta \cos A)(\theta - N, s') \]  \hspace{1cm} (7)

\[ (1 + \frac{4R}{3}) \theta'' + \frac{1}{2} f'' + N_b \theta' + N_i \theta^2 = \xi \left( S_f f'' + f' \frac{\partial f}{\partial \xi} - \theta \frac{\partial f}{\partial \xi} \right) \]  \hspace{1cm} (8)

\[ s'' + \frac{1}{2} Lef s' + \frac{N_i}{N_b} \theta' = \xi \left( f' \frac{\partial f}{\partial \xi} - \theta \frac{\partial f}{\partial \xi} \right) \]  \hspace{1cm} (9)

where

\[ Ra = \left(1 - \phi \right) \rho f x_b \rho_k (T_w - T_0) \] is the Rayleigh number

\[ Gr = \frac{(1 - \phi) \rho f x_b \rho_k (T_w - T_0)}{\mu^2} \] is the Grashof number

\[ R = \frac{4\gamma \zeta^3}{KK_e} \] is the Radiation parameter

\[ N = \frac{\left( \rho_p - \rho f \right) h (T_w - T_0)}{\rho f \alpha f (T_w - T_0) (1 - \phi)} \] is the buoyancy ratio

\[ N_b = \frac{\alpha_b f (T_w - T_0)}{\alpha f (1 - \phi)} \] is the Brownian motion parameter

\[ N_l = \frac{\alpha b f (T_w - T_0)}{\alpha f (1 - \phi)} \] is the thermophoresis parameter

\[ Le = \frac{\alpha D}{D} \] is the Lewis number and

\[ S_f = \frac{B l}{T_w - T_{0,0}} \] is the thermal stratiﬁcation parameter.

The associated boundary conditions are

\[ f + 2\frac{\partial f}{\partial \xi} = 0, \ \theta = 1 - S_f \zeta, s = 1 \] at \( \eta = 0 \) \hspace{1cm} (10a)

\[ f' = 0, \ \theta \rightarrow 0, \ s \rightarrow 0 \ \text{as} \ \eta \rightarrow \infty \] \hspace{1cm} (10b)

The primary objective of this study is to estimate the parameters of engineering interest in fluid flow, heat and nanoparticle mass transport processes, namely the Nusselt number \( Nu_{c} \), and nanoparticle Sherwood number \( NS_{hu} \). These parameters characterize the wall heat and nanoparticle mass transfer rates, respectively.

The dimensionless local Nusselt number and the nanoparticle Sherwood number are given by

\[ \frac{Nu_{c}}{\sqrt{Ra_{c}}} \left( \frac{1 + 4R/3}{1 - S_f \eta} \right) \rho f (T_w - T_0) = \frac{s'(\xi, 0)}{\sqrt{1 + \delta^2}} \] \hspace{1cm} (11)

III. METHOD OF SOLUTION

To solve the system of Eqs. (7) – (9) along with the boundary conditions (10), we first apply a local similarity and non-similarity method which has been applied by the many of the researchers [14, 15] to solve various non-similar boundary value problems. The boundary value problems resulting from this method are solved by the successive linearization method.

In the first level of truncation, the terms accompanied by \( \zeta(\xi) \) are assumed to be very small. This is particularly true when \( \zeta < 1 \). Thus the terms with \( \zeta(\xi) \) in Eqs. (7) – (9) can be neglected to get the following system of equations.

\[ \frac{2Gr}{\sqrt{1 + \delta^2}} = f^* f^* = (\sin A + \delta \cos A)(\theta - N, s') \]  \hspace{1cm} (12)

\[ (1 + \frac{4R}{3}) \theta'' + \frac{1}{2} f'' + N_b \theta' + N_i \theta^2 = \xi \left( S_f f'' + f' \frac{\partial f}{\partial \xi} - \theta \frac{\partial f}{\partial \xi} \right) \]  \hspace{1cm} (13)

\[ s'' + \frac{1}{2} Lef s' + \frac{N_i}{N_b} \theta' = \xi \left( f' \frac{\partial f}{\partial \xi} - \theta \frac{\partial f}{\partial \xi} \right) \]  \hspace{1cm} (14)

The associated boundary conditions are

\[ f = 0, \ \theta = 1 - S_f \zeta, s = 1 \] at \( \eta = 0 \) \hspace{1cm} (15a)

\[ f' = 0, \ \theta \rightarrow 0, \ s \rightarrow 0 \ \text{as} \ \eta \rightarrow \infty \] \hspace{1cm} (15b)

For the second level of truncation we introduce \( g = \delta / \zeta, h = \partial \theta / \zeta, \) and \( k = \partial / \zeta \) and recover the neglected terms at the first level of truncation. Thus the governing equations at the second level reduces to

\[ f^* + \frac{2Gr}{\sqrt{1 + \delta^2}} = f^* f^* = (\sin A + \delta \cos A)(\theta - N, s') \]  \hspace{1cm} (16)

\[ (1 + \frac{4R}{3}) \theta'' + \frac{1}{2} f'' + N_b \theta' + N_i \theta^2 = \xi \left( S_f f'' + f' \frac{\partial f}{\partial \xi} - \theta \frac{\partial f}{\partial \xi} \right) \]  \hspace{1cm} (17)

\[ s'' + \frac{1}{2} Lef s' + \frac{N_i}{N_b} \theta' = \xi \left( f' \frac{\partial f}{\partial \xi} - \theta \frac{\partial f}{\partial \xi} \right) \]  \hspace{1cm} (18)

The associated boundary conditions are

\[ f + 2\frac{\partial f}{\partial \xi} = 0, \ \theta = 1 - S_f \zeta, s = 1 \] at \( \eta = 0 \) \hspace{1cm} (19a)

\[ f' = 0, \ \theta \rightarrow 0, \ s \rightarrow 0 \ \text{as} \ \eta \rightarrow \infty \] \hspace{1cm} (19b)

At the third level of truncation we differentiate Eqs. (16) – (19) with respect to \( \zeta \) and neglect the terms \( \partial / \zeta^2, \partial / \zeta \) and \( \partial / \zeta \) get to the following system of equations

\[ g'' + 2Gr(1 + \delta^2)^{-1} \delta \theta' + 2Gr(1 + \delta^2)^{-1} 2 g'' + 2Gr(1 + \delta^2)^{-1} 2 f'' = \xi \left( \frac{\sin A + \delta \cos A}{(\delta - N, s')} \right) \]  \hspace{1cm} (20)

\[ (1 + \frac{4R}{3}) \theta'' + \frac{1}{2} f'' + N_b \theta' + N_i \theta^2 = \xi \left( S_f f'' + f' \frac{\partial f}{\partial \xi} - \theta \frac{\partial f}{\partial \xi} \right) \]  \hspace{1cm} (21)

\[ s'' + \frac{1}{2} Lef s' + \frac{N_i}{N_b} \theta' = \xi \left( f' \frac{\partial f}{\partial \xi} - \theta \frac{\partial f}{\partial \xi} \right) \]  \hspace{1cm} (22)

The associated boundary conditions are

\[ g = 0, \ h = - S_f, k = 0 \] at \( \eta = 0 \) \hspace{1cm} (23a)

\[ g' = 0, \ h = - S_f, k = 0 \] \hspace{2cm} (23b)

The set of differential equations (16) – (18) and (20) – (22) together with the boundary conditions (19) and (23) are solved using successive linearization method [16]-[18]. Using this method the non linear boundary layer equations reduce to a system of linear differential equations. The Chebyshev pseudo spectral method is then used to transform the iterative sequence of linearized differential equations into a system of linear algebraic equations which are converted into a matrix system.

In this method we assume that the independent variables \( f(\eta), \theta(\eta), s(\eta), g(\eta), h(\eta) \) and \( k(\eta) \) can be expressed as

\[ \{ f(f(\eta), \theta(\eta), s(\eta), g(\eta), h(\eta), k(\eta) \} \]  \hspace{1cm} (24)

\[ \{ f_0(\eta), \theta_0(\eta), s_0(\eta), g_0(\eta), h_0(\eta), k_0(\eta) \} \]  \hspace{1cm} (25)

\[ \{ f_{i_0}(\eta), \theta_{i_0}(\eta), s_{i_0}(\eta), g_{i_0}(\eta), h_{i_0}(\eta), k_{i_0}(\eta) \} \]  \hspace{1cm} (26)

\[ \{ f_{n+1}(\eta), \theta_{n+1}(\eta), s_{n+1}(\eta), g_{n+1}(\eta), h_{n+1}(\eta), k_{n+1}(\eta) \} \]  \hspace{1cm} (27)

\[ \{ f_{n+1}(\eta), \theta_{n+1}(\eta), s_{n+1}(\eta), g_{n+1}(\eta), h_{n+1}(\eta), k_{n+1}(\eta) \} \]  \hspace{1cm} (28)

where \( f_0, \theta_0, s_0, g_0, h_0 \) and \( k_0 \) are the approximations which are obtained by recursively solving the linear part of the equation system that results from substituting (24) in (16).
The approximate solutions for \( f(\eta), \theta(\eta), \psi(\eta), \phi(\eta), h(\eta) \) and \( k(\eta) \) are then obtained as

\[
M \sum_{m=0}^{M} f_m(\eta), \theta_m(\eta), \psi_m(\eta), \phi_m(\eta), h_m(\eta), k_m(\eta) \]

where \( M \) is the order of SLM approximation. The linearized equations were solved using the Chebyshev spectral collocation method [20]. The unknown functions are approximated by the Chebyshev interpolating polynomials in such a way that they are collocated at the Gauss-Lobatto points defined as

\[
\xi_j = \cos \left( \frac{j \pi}{N} \right), \quad j = 0, 1, 2, ..., N
\]

where \( N \) is the number of collocation points used. The physical region \([0, \infty)\) is transformed into the region \([-1, 1]\) using the domain truncation technique in which the problem is solved on the interval \([0, L]\) instead of \([0, \infty)\). This leads to the mapping

\[
\eta = \frac{\xi + 1}{2}, \quad -1 \leq \xi \leq 1
\]

where \( L \) is a scaling parameter used to invoke the boundary condition at infinity. The functions \( f, \theta, \psi, \phi, h, \) and \( k \), are approximated at the collocation points by

\[
f_j(\xi), \theta_j(\xi), \psi_j(\xi), \phi_j(\xi), h_j(\xi), k_j(\xi) = \sum_{k=0}^{N} f_k(\xi) \]

\[
\theta_j(\xi) = \psi_j(\xi) = \phi_j(\xi) = h_j(\xi) = k_j(\xi) = T_k(\xi)
\]

where \( T_k(\xi) \) is the \( k^{th} \) Chebyshev polynomial defined by

\[
T_k(\xi) = \cos[k \cos^{-1} \xi]
\]

The derivatives of the variables at the collocation points are represented as

\[
\frac{d^m}{d\eta^m}(\bullet)_j = \sum_{k=0}^{N} 2 \frac{D^m_k(\bullet)}{L} T_k(\xi)
\]

where \( a \) is the order of differentiation and \( D \) being the Chebyshev spectral differentiation matrix. Substituting Eqs. (28) – (30) into linearized form of equations leads to the matrix equation

\[
A \cdot X_{i+1} = R_{i+1}
\]

In Eq. (31), \( A \cdot \) is a \((6N + 6) \times (6N + 6)\) square matrix and \( X_i \) and \( R_i \) are \((6N + 1) \times 1\) column vectors defined by

\[
A = [A_{pq}], \quad X = [X_p]^T \quad \text{and} \quad R = [r_{q,i}], \quad p,q = 1,2, ..., 6
\]

After modifying the matrix system (32) to incorporate boundary conditions, the solution is obtained as

\[
X_{i+1} = \text{R}_{i+1} A^{-1}_{i+1}
\]

IV. RESULTS AND DISCUSSION

Table I shows the comparison of the results of the values of \( \theta'(0) \) and \( f'(0) \) of the present paper for different values of Gr and fixed values of \( A=\pi/2, \alpha=0, \) \( S_T=0 \) with the results obtained by Plumb and Huenefeld [13]. It is shown that these two results are in excellent agreement.

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<th>( f'(0) )</th>
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<td>( \text{Plumb &amp; Huenefeld} )</td>
<td>( \text{Present} )</td>
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The effect of the wave amplitude on the local Nusselt number \( Nu_{\xi}(1-S_T)\xi/Ra_z^{1/2} \) and nanoparticle Sherwood number \( \text{NSh}_{\xi}/Ra_z^{1/2} \) is plotted in Figs. 1 - 2. This figure reveals that an enhancement in wavy amplitude decreases the local heat and nanoparticle mass transfer rates. In general, we conclude that the surface becomes more roughened for increasing values of amplitude of the wavy surface.

The variation of heat and nanoparticle mass transfer rates for various values of the angle of inclination \( \alpha \) and thermal stratification parameter \( S_T \) is displayed in Figs. 3 - 4. This figure shows that increasing the angle of inclination increases the buoyancy force and assist the flow, leading to an increase in the heat and nanoparticle mass transfer rates but an increase in the value of thermal stratification parameter \( S_T \) reduces the both heat and nanoparticle mass transfer rates.

The effect of Brownian motion parameter \( N_b \) and thermophoresis parameter \( N_t \) on the heat and nanoparticle mass transfer rates is presented in Figs. 5 - 6. It is shown dimensionless heat transfer rate decreases with increase in both the Brownian motion parameter and thermophoresis parameter. An increase in the value of Brownian motion parameter enhances the nanoparticle mass transfer rate and an increase in the value of thermophoresis parameter reduces the nanoparticle mass transfer rate.

Figs. 7 - 8 display the streamwise distribution of Nusselt and nanoparticle Sherwood numbers for different values of radiation parameter \( R \) and non-Darcy parameter \( Gr \). It is seen that the heat and nanoparticle mass transfer rates enhance with increase in the radiation parameter and reduces with increase in the value of Grashof number.
Fig. 2 Effect of wave amplitude $a$ on heat transfer rate

Fig. 3 Effect of the wave amplitude $a$ on the nanoparticle mass transfer rate

Fig. 4 Effect of angle of inclination $A$ and Thermal stratification parameter $S_T$ on heat transfer rate

Fig. 5 Effect of angle of inclination $A$ and Thermal stratification parameter $S_T$ on the nanoparticle mass transfer rate

Fig. 6 Effect of Brownian motion parameter $N_b$ and Thermophoresis parameter $N_t$ on heat transfer rate

Fig. 7 Effect of Brownian motion parameter $N_b$ and Thermophoresis parameter $N_t$ on nanoparticle mass transfer rate

Fig. 8 Effect of Grashof number $Gr$ and Radiation parameter $R$ on heat transfer rate

Fig. 9 Effect of Grashof number $Gr$ and Radiation parameter $R$ on nanoparticle mass transfer rate
REFERENCES


