Nomograms for Synthesizing Crank Rocker Mechanism with a Desired Optimum Range of Transmission Angle

Dr. Khaled M. Khader

Abstract—The transmission angle is a significant criterion of mechanisms design. It's the smaller angle between the coupler link and the follower (output) link of a planar mechanism. Transmission angle value which equal to 90° provides the best characteristics to the mechanism motion, while this motion becomes very hard with values nearly 0° or 180°. As well as, the mechanism transmission motion may be impossible at 0° or 180°. Furthermore, the force-transmission characteristics of a crank rocker mechanism can be judged by concerning the transmission angle range through a whole crank rotation. Obviously, design nomograms which is presented in this paper can be consider as an effective fast tool helps the designers for direct selection of the suitable mechanism links ratios providing a definite synthesized transmission angle range.

Keywords—Transmission angle, Mechanism, Synthesis, Design techniques.

I. INTRODUCTION

The transmission angles of a planar crank rocker mechanism can be defined as the smaller angle between the coupler link and the follower rocker link as in [1]. It can be also defined as the smaller angle between the direction of absolute velocity vector of rocker link and direction of velocity difference vector of driving link at the connection joint as in [2]. It is well known that, the main objectives of mechanisms designers are low input torque variation, optimum transmission angle, smooth transmission, low periodic loads in bearing and fewer vibrations. Moreover, the transmission angle plays an important role for providing a good or bad mechanism motion’s characteristics. Hence, a reasonable optimal transmission angle can treat the most problems in mechanisms as in [3]. The low fluctuation of torque can't be always guaranteed with a large value of transmission angle. Also, through a large error motion, very small or very large transmission angle outcomes undesirable noisy mechanism as mentioned in [4].

Unfortunately, optimizing the mechanism linkages with considering their force transmission characteristics simultaneously with synthesizing the transmission angle is a complicated problem. On the other hand, this optimizing problem can be considerably simplified if the transmission angle range is restricted to be bounded by a two specific limits as in [5].

Clearly, the maximum and minimum transmission angles are highly depending on the rocker angle. Moreover, the reasonable deviation of the transmission angle value from 90° is significant for providing a good characteristics to mechanism motion and can guarantee a smooth motion without undesirable vibrations at high speed as in [3], [6]. On the other hand, the large transmission angle variation plays an effective role in reducing the effectiveness of force transmission where mechanisms having a wide deviation ranges of transmission angle from 90°, reveal undesired operational characteristics at high speed like noise and jerk as in [2], [7]. The mechanisms designers are always looking for designing mechanisms with a suitable linkages which have transmission angle range is as small as possible, especially in the existence of significant joint friction as in [1], [3].

Synthesis of four bar mechanism for polynomial function generation dealing with maximum and minimum transmission angles is presented in [8]. In addition to, an analytical synthesis of crank rocker mechanisms dealing with the motion between two small separated positions of minimum and maximum transmission angles is presented in [9]. Also, a shape optimization for path synthesis of crank rocker mechanisms dealing with optimal transmission angle is presented in [10]. Moreover, synthesis of planar four bar mechanism is presented in [11] for exact three coupler positions which can be attained with transmission angles less than 18 % of the optimum value of 90°.

As well as, [12] presented a new method of designing four bar function generators with a good effective transmission angle which can be considered an optimum angle, in a minimax sense, when their extreme values have variations are equally around 90°. Solved example for analytical optimizing four bar transmission angle is presented in [13] through supposing the dimension of coupler link in addition to dimensions ranges of fixed, crank and rocker links.

Furthermore, transmission angle range 30°-150° is recommended in [14]. As well as, some different transmission angle ranges are suggested like 35°-145°; 40°-140°; 45°-135° as in [6].

A few publications are dealing with selecting the transmission angle of crank rocker mechanisms using the graphical techniques. The minimum transmission angle can be selected from design charts depending on the value of angle

Khaled Khader is member of the teaching staff with the Department of Production Engineering & Mechanical Design, Faculty of Engineering, University of Menoufia, Shebin El-kom, Menoufia, Egypt. (corresponding author’s phone: 00201223538574 ; fax 0020482235695; e-mail: khkhmo2@hotmail.com)
between two dead-center positions of the rocker and the corresponding angle of the crank in addition to assuming three linkages dimensions as in [15], [16]. Hence, the design nomograms which is presented in this paper can be consider as a fast tool for directly selecting the suitable ratios of mechanism links lengths which give a definite synthesized transmission angle range.

II. TRANSMISSION ANGLE

The transmission angle (μ) of a planar crank rocker mechanism (O_1ABo_2) is indicated in Fig. 1 as follows;

$$
\mu = \cos^{-1}\left(\frac{r_3^2 + r_4^2 - (r_1 - r_2)^2}{2r_3r_4}\right)
$$

(1)

Where; fixed, crank, coupler and rocker links lengths are r_1, r_2, r_3 and r_4, respectively.

Furthermore, the transmission angle (μ) of the crank rocker mechanism can be formulated as follows;

$$
\mu = \cos^{-1}\left(\frac{r_3^2 + r_4^2 - (r_1 - r_2)^2}{2r_3r_4} \cos \psi\right)
$$

III. MINIMUM AND MAXIMUM TRANSMISSION ANGLE

Maximum transmission angle (μ_max) and the minimum transmission angle (μ_min) of a planar crank rocker mechanism is shown in Fig. 2 as follows;

$$
\mu_{\text{max}} = \cos^{-1}\left(\frac{r_3^2 + r_4^2 - (r_1 + r_2)^2}{2r_3r_4}\right)
$$

(2)

$$
\mu_{\text{min}} = \cos^{-1}\left(\frac{r_3^2 + r_4^2 - (r_1 - r_2)^2}{2r_3r_4}\right)
$$

(3)

IV. THEORETICAL FORMULATIONS OF DESIRED NOMOGRAMS

Clearly, transmission angle can be considered as the optimal one when this angle has maximum and minimum values which have variations equally around 90° as in [12]. Hence, the desired design nomograms (charts) of optimal transmission angle are using the meaning of μ_min=(π/2)-δ and μ_max=(π/2)+δ, where δ can be assumed related to the designer requirements. Thus, the summation of μ_min and μ_max can be equal to (π). Hence, cosines of μ_min and μ_max can be written as follows;

$$
\cos \mu_{\text{min}} + \cos \mu_{\text{max}} = 0
$$

(4)

Also, the pervious equation can be rewritten as follows;

$$
\frac{r_3^2 + r_4^2 - (r_1 - r_2)^2}{2r_3r_4} + \frac{r_3^2 + r_4^2 - (r_1 + r_2)^2}{2r_3r_4} = 0
$$

(5)

As well as, the pervious equation can be simplified as follows;

$$
\frac{r_3^2 + r_4^2 - r_1^2}{2r_3r_4} = 1
$$

(6)

Clearly, using linkages lengths ratios instead of using linkages lengths can simplify the designers work. Thus, the pervious equation can be rewritten as follows;

$$
R_2^2 = R_3^2 + R_4^2 - 1
$$

(7)

Where, ratios R_2, R_3 and R_4 can be written as follows;

$$
R_2 = \frac{r_2}{r_1}, \quad R_3 = \frac{r_3}{r_1} \quad \text{and} \quad R_4 = \frac{r_4}{r_1}
$$

(8)

The desired charts (nomograms) will depend on assuming only two values. The first assumed value is the desired rocker angle (ϕ_I) which occurs at the minimum transmission angle and the second one is the desired transmission angle range through supposing value of (δ). These two angles (ϕ_I & δ) in addition to coupler angle (ο_I) are indicated in Fig. 3 which shows the rocker angle (ϕ_I) at the minimum transmission angle μ_min as follows;

$$
\mu_{\text{min}} = \frac{\pi}{2} - \delta = \phi_I - \mu_{\text{min}} + \frac{\pi}{2}
$$

(9)

The triangle sine rule can be applied for the triangle ABO_2 which is indicated in Fig. 3 as in [18]. Hence, the three fraction equalities can be shown as follows;
\[
\frac{r_1 - r_2}{\sin \mu_{\min}} = \frac{r_3}{\sin (\pi - \phi)} = \frac{r_4}{\sin \theta_1}
\]

(9)

The pervious equation can be rewritten using the linkages ratios as follows;
\[
\frac{1 - R_2}{\sin \mu_{\min}} = \frac{R_3}{\sin (\phi - \mu)} = \frac{R_4}{\sin \phi_1}
\]

(10)

Hence, the pervious equation can be written again as follows;
\[
\frac{1 - R_2}{\sin \mu_{\min}} = \frac{R_3}{\sin \phi_1} = \frac{R_4}{\sin (\phi_1 - \mu_{\min})}
\]

(11)

Where, \(\sin (\pi - \phi) = \sin \phi_1\) and \(\phi = \phi_1 - \mu_{\min}\). Thus, a new equation can be written form the pervious equation as follows;
\[
R_3 = \frac{\sin \phi_1}{\sin \mu_{\min}} (1 - R_2)
\]

(12)

Also, the pervious equation can be rewritten as follows;
\[
R_3 = A(1 - R_2) \quad \text{where} \quad A = \frac{\sin \phi_1}{\sin \mu_{\min}}
\]

(13)

As well as, a new equation can be written form Eq. (11) as follows;
\[
R_4 = \frac{\sin (\phi_1 - \mu_{\min})}{\sin \mu_{\min}} (1 - R_2)
\]

(14)

Also, the pervious equation can be rewritten as follows;
\[
R_4 = B(1 - R_2) \quad \text{where} \quad B = \frac{\sin (\phi_1 - \mu_{\min})}{\sin \mu_{\min}}
\]

(15)

Both \(R_3\) and \(R_4\) which appeared in (7) can be substituted with another forms which appeared in (13) and (15) respectively. Equation (7) can be written in new form as follows;
\[
R_2^2 = A^2(1 - R_2)^2 + B^2(1 - R_2)^2 - 1
\]

(16)

The pervious equation of \(R_2\) can rewritten as follows;
\[
(A^2 + B^2 - 1)R_2^2 - 2(A^2 + B^2)R_2 + (A^2 + B^2 - 1) = 0
\]

(17)

Thus;
\[
R_2 = \frac{A^2 + B^2}{A^2 + B^2 - 1} \pm \sqrt{A^2 + B^2 - 1}
\]

(18)

It is well known that, the crank is shorter than the fixed link of the crank rocker mechanism, means that \(R_2\) less than one.

Thus, the plus sign of square root in (18) can be rejected because of the value \((A^2+B^2)/(A^2+B^2-1)\) is greater than one. Hence, the pervious equation can be reduced to be;
\[
R_2 = \frac{A^2 + B^2}{A^2 + B^2 - 1} - \frac{\sqrt{A^2 + B^2 - 1}}{A^2 + B^2 - 1}
\]

(19)

Now, the value \(R_2\) can be calculated using the pervious equation. Also, the values \(R_3\) and \(R_4\) can be calculated using (13) and (15) respectively. These three values \(R_2, R_3\) and \(R_4\) are only depending on the two values \((A & B)\) which are only depending on the values of \((\phi_1 & \mu_{\min})\) as follows;
\[
A = \frac{\sin \phi_1}{\sin \mu_{\min}} \quad \text{and} \quad B = \frac{\sin (\phi_1 - \mu_{\min})}{\sin \mu_{\min}}
\]

(20)

Furthermore, these three ratios \(R_2, R_3\) and \(R_4\) are only the functions of \(\phi_1\) and \(\delta\). Hence, design charts (nomograms) can be drawn easily for synthesizing the linkages ratios \(R_2, R_3\) and \(R_4\) of crank rocker mechanism through assuming the value of \(\delta\) which determine the minimum transmission angle \((\mu_{\min})\) and assuming the value of rocker angle \((\phi_1)\) which corresponds \((\mu_{\min})\).

It is well known that, the time ratio \((T_r)\) is considered as an important criteria through designing the crank rocker mechanism. Also, satisfying the condition \((\mu_{\min} + \mu_{\max} = \pi)\) can make the time ratio is unity through designing the crank rocker mechanism as in [19]. The time ratio \((T_r)\) can be formulated as follow;
\[
T_r = \frac{\psi_2 - \psi_1}{360 - (\psi_2 - \psi_1)}
\]

(21)

Where;
\[
\psi_1 = \cos^{-1}\left(\frac{1 + (R_2 + R_3)^2 - R_4^2}{2(R_2 + R_3)}\right)
\]

(22)

and;
\[
\psi_2 = \pi + \cos^{-1}\left(\frac{1 + (R_3 - R_2)^2 - R_4^2}{2(R_3 - R_2)}\right)
\]

(23)

Rocker angle \((\phi_II)\) corresponds \((\mu_{\max})\) can be formulated as;
\[
\phi_II = \pi + \cos^{-1}\left(\frac{(1 + R_2^2 + R_4^2 - R_3^2)}{2(1 + R_2)R_4}\right)
\]

(24)

Clearly, the rocker angle \((\phi)\), rocker angular velocity \((\phi')\) and rocker angular acceleration \((\phi'')\) are important factors of mechanism performance. Solving the equation of rocker angle \((\phi)\) which is a function of \((\psi, r_1, r_2, r_3\) and \(r_4)\) is presented as two possible solutions in [20] which can be reformulated as follows;
\[
F_0 = F_1 \cos \phi + F_2 \sin \phi
\]

(25)

Where;
\[
F_0 = 1 + R_2^2 + R_4^2 - R_3^2 - 2R_2 \cos \psi
\]

(26)

\[
F_1 = -2R_4 (1 - R_2 \cos \psi)
\]

\[
F_2 = -2R_2 R_4 \sin \psi
\]

Then, the \(\sin (\phi)\) and \(\cos (\phi)\) can be formulated as follows;
\[
\sin \phi = \left(\frac{F_0 F_2}{F_1^2 + F_2^2}\right) - \left(\frac{F_2 \sqrt{F_1^2 + F_2^2 - F_0^2}}{F_1^2 + F_2^2}\right)
\]

(27)

\[
\cos \phi = \left(\frac{F_2 \sqrt{F_1^2 + F_2^2 - F_0^2}}{F_1^2 + F_2^2}\right) + \left(\frac{F_0 F_1}{F_1^2 + F_2^2}\right)
\]

(28)

Thus, the rocker angle \((\phi)\) can be calculated through comparing the values and signs of \(\sin (\phi)\) and \(\cos (\phi)\) which are shown in the previous equation. As well as, rocker angular velocity \((\phi')\) can be found by derivation of (25).

V. NOMOGRAMS (CHARTS) FOR SYNTHESIZING OF CRANK ROCKER MECHANISM

It is easy now to draw three nomograms (charts) as an easy tools in order to help the mechanisms designers where they can directly select the suitable mechanism links ratios \((R_2, R_3\) and \(R_4)\) which give them a definite synthesized transmission angle.
These charts can be drawn through assuming different values of rocker angle \( \phi_I \) and assuming different values of \( \delta \) which give corresponding different values of minimum transmission angles \( \mu_{\text{min}} \). Hence, substituting the assumed values \( (\phi_I, \mu_{\text{min}}) \) in (13), (15) and (19) in order to calculate the linkages ratios \( R_2, R_3 \) and \( R_4 \). The relation between \( R_3 \) and the rocker angle \( \phi_I \) which corresponds to the desired minimum transmission angle \( \mu_{\text{min}} \) is shown in Fig. 4 through different curves of different values of \( \delta \).

![Fig. 4 Relation between \( R_3 \) and the rocker angle \( \phi_I \) which corresponds to minimum transmission angle \( \mu_{\text{min}} \)](image)

As well as, the relation between \( R_3 \) and the rocker angle \( \phi_I \) which corresponds to the desired minimum transmission angle \( \mu_{\text{min}} \) is shown in Fig. 5 through different curves of different values of \( \delta \).

![Fig. 5 Relation between \( R_3 \) and rocker angle \( \phi_I \) which corresponds to minimum transmission angle \( \mu_{\text{min}} \)](image)

Also, the relation between \( R_4 \) and the rocker angle \( \phi_I \) which corresponds to the desired minimum transmission angle \( \mu_{\text{min}} \) is shown in Fig. 6 through different curves of different values of \( \delta \).

![Fig. 6 Relation between \( R_4 \) and rocker angle \( \phi_I \) which corresponds to minimum transmission angle \( \mu_{\text{min}} \)](image)

Furthermore, a compact chart which is shown in Fig. 7 presents the relation between \( (R_2, R_3 \& R_4) \) and the rocker angle \( \phi_I \) which corresponds to desired minimum transmission angle \( \mu_{\text{min}} \) through different curves of different values of \( \delta \). These charts can be used through drawing a vertical line from the assumed rocker angle \( \phi_I \) to intersect the curve of assumed value of \( \delta \), hence drawing a horizontal line till the desired linkages ratios.

![Fig. 7 Relation between \( (R_2, R_3 \& R_4) \) and rocker angle \( \phi_I \) of \( \mu_{\text{min}} \)](image)

VI. RESULTS AND DISCUSSIONS

As example of using the pervious nomogrames for synthesizing linkages ratios \( (R_2, R_3 \& R_4) \) of crank rocker mechanism or using (13), (15) and (19) for calculating the accurate values of linkages ratios.

Assume a mechanism designer select the desired rocker angle \( \phi_I=90^\circ \) which corresponds to desired minimum transmission angle \( \mu_{\text{min}}= 60^\circ \) and he also select the desired value of \( \delta = 30^\circ \). Hence, the ratios \( (R_2, R_3, \& R_4) \) can be
determined as; $R_2=0.2086$, $R_3=0.9136$ and $R_4=0.4570$. By substituting the previous values of $(R_2, R_3$ and $R_4)$ in (2) and (3), the calculated values of minimum and maximum transmission angles are; $(\mu_{\text{min}}=60^\circ)$ and $(\mu_{\text{max}}=120^\circ)$ which are the same as the assumed desired values. As well as, results of six different mechanism are listed in Table I.

<table>
<thead>
<tr>
<th>Nb.</th>
<th>$\psi_1$ (deg.)</th>
<th>$\delta$ (deg.)</th>
<th>$\mu_{\text{min}}$ (deg.)</th>
<th>$R_2$</th>
<th>$R_3$</th>
<th>$R_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>85</td>
<td>35</td>
<td>55</td>
<td>0.2435</td>
<td>0.9198</td>
<td>0.4618</td>
</tr>
<tr>
<td>2</td>
<td>90</td>
<td>30</td>
<td>60</td>
<td>0.2086</td>
<td>0.9136</td>
<td>0.4570</td>
</tr>
<tr>
<td>3</td>
<td>120</td>
<td>25</td>
<td>65</td>
<td>0.2211</td>
<td>0.7438</td>
<td>0.7040</td>
</tr>
<tr>
<td>4</td>
<td>140</td>
<td>20</td>
<td>70</td>
<td>0.1633</td>
<td>0.5715</td>
<td>0.8367</td>
</tr>
<tr>
<td>5</td>
<td>135</td>
<td>15</td>
<td>75</td>
<td>0.1286</td>
<td>0.6372</td>
<td>0.7813</td>
</tr>
<tr>
<td>6</td>
<td>135</td>
<td>10</td>
<td>80</td>
<td>0.0862</td>
<td>0.6554</td>
<td>0.7602</td>
</tr>
</tbody>
</table>

Also, by substituting the values of $(R_2, R_3, \text{and } R_4)$ in (21), (22), (23) and (24) for calculating the values of $(\psi_1, \psi_2, T_1$ and $\phi_H)$. The calculated values are; $\psi_1=23.98^\circ$, $\psi_2=203.98^\circ$, $T_1=1$ and $\phi_H=139.1^\circ$.

The transmission angle $(\mu)$ and the rocker angle $(\phi)$ can be calculated using (1), (26) and (27) for each crank angle $(\psi)$. The relation between $(\psi)$ and both $(\mu$ & $\phi)$ is shown in Fig. 8.

As well as, relation between $(\psi)$ and rocker angular velocity ratio is shown in Fig. 9 where the rocker angular velocity ratio equals to the rocker angular velocity $(\omega_o)$ divided by crank angular velocity $(\omega_c)$. The curve in Fig. 9 of rocker angular velocity ratio indicates a symmetric values of this ratio above and down zero axis where the mechanism time ratio is unity.

**VII. CONCLUSION**

The transmission angle is a significant criterion of mechanisms design where transmission angle value which equal to $90^\circ$ provides the best characteristics to the mechanism motion, while this motion becomes very hard with values nearly $0^\circ$ or $180^\circ$. The nomograms which presented here are dealing with a good effective transmission angle which can be considered an optimum angle, where its extreme values have variations equally around $90^\circ$. These nomograms (charts) are only depending on the value of desired minimum transmission angle $(\mu_{\text{min}})$ and the value of the desired rocker angle $(\phi_{\text{H}})$ which corresponds to the desired minimum transmission angle.

Obviously, the presented design nomograms can be considered as an effective fast tool which helps the designers for directly selection of suitable mechanism links ratios giving a definite synthesized transmission angle range.

**REFERENCES**


